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Abstract

A non-linear model is applied where suddenly strong spurts of exports occur when changes of the exchange rate go beyond a zone of inaction. We call the latter a “play” area – analogous to mechanical play and implement an algorithm describing path-dependent play-hysteresis into a regression framework. The hysteretic impact of real exchange rates on German exports is then estimated based on quarterly data from 1995Q1 to 2010Q3. For some of the main export partners of Germany outside the euro area and some of the most important tradable sectors we find significant hysteretic effects for a part of the German exports.

JEL-Classification: F14, C51

Keywords: Exchange rate movements, play-hysteresis, modelling techniques, switching / spline regression, export demand

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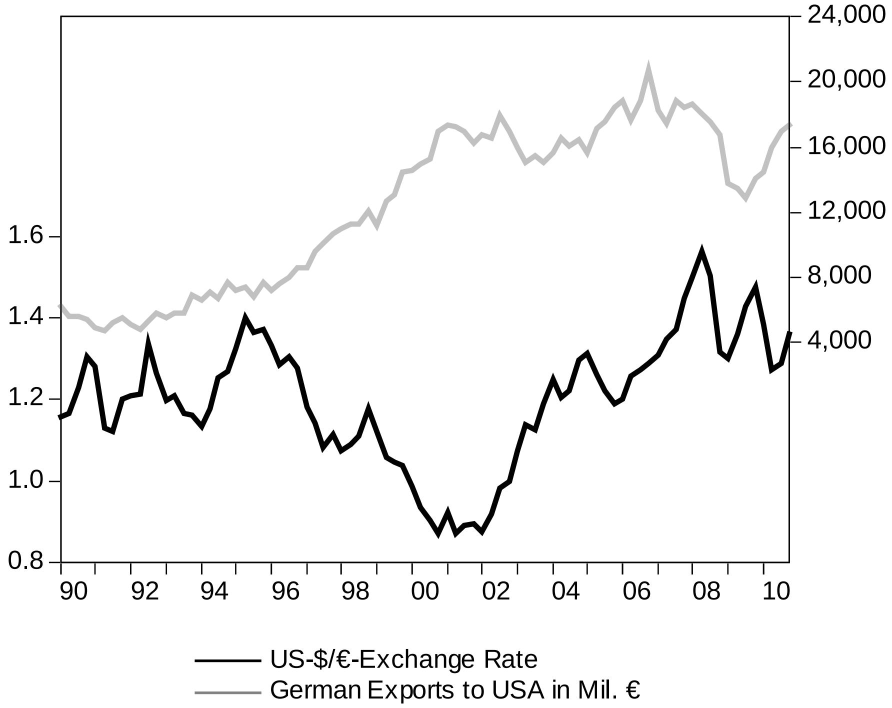
1. Introduction

European politicians and business persons are frequently concerned with the external value of the European currency. In fact, concerns are expressed nearly every time when the euro appreciates. For example by BusinessEurope President Ernest-Antoine Seilliere who in 2007 said to Jean-Claude Juncker, the chairman of Eurogroup, that he also agreed that the euro exchange rate had reached a “pain threshold“ for European companies (*Dow Jones International News* 2007). This statement implies that beyond some boundaries (“pain threshold”) stronger export reactions in case of an exchange rate are expected.

A closer look into the more recent episode in which the $/€ rate reached “all-time highs” might give a first indication in this regard. According to Figure 1, from 1999 to 2001 a monotonously ongoing €-depreciation is accompanied with growing German exports to the US. The reversal of this exchange rate movement seems at first (around 2002-2003) to have only a limited effect on the exports. However, later the ongoing one-directional further appreciation around 2003-2004 seems to lead to a more significant negative effect on the exports. At a first glance, exports seem to show a limited reaction on small and temporary exchange rate variations (e.g. 1997-1999 and 2002-2003), but to react more significantly to one-directional and ongoing movements of the exchange rate (e.g. in the depreciation period from 1999 to 2002, and in an appreciation period around 2003-2005).

Figure 1 – *$/€ exchange rate and German exports to the US*



Source: Own calculation based on Eurostat and Bundesbank data.

What are potential reasons of a *weak reactions* of German exports to small exchange rate movements with a varying direction?

Hedging of exchange rate uncertainty: In the short run, i.e. in the case of an only transitory appreciation of the euro, the choice of the invoice currency and the extent of cross-currency hedging plays a role. Three quarters of all foreign currency receivables from export business are hedged against exchange rate related losses for some time. However, hedging cushions the appreciation pressure only for a limited period (Deutsche Bundesbank, 2008).

German export product line and price elasticity of exports: The share of relatively price-inelastic goods in the range of German exports is quite high. Exports to non-euro area countries, in particular, respond weakly to price competitiveness (Deutsche Bundesbank 2008). Even more important: making up for a share of around 46.8 % in 2009[1] machinery, equipment and vehicles dominate Germany's industrial production. German firms are often highly specialized in these areas and in terms of technology maintained their position as the world market leader. As a consequence, importers are not able to or even do not want to switch to other suppliers even when the external value of the € increases because switching costs would be too high for them.

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[1] Own calculations based on Eurostat data. The shares are derived as an average of 2008q1 to 2010q3.

Pricing-to-market by German exporting firms: German export prices show a weak cost pass-through due to a *pricing-to-market* strategy. This implies that a strong € is mainly absorbed through a reduction in the profit margin (Stahn 2007).

Sunk market entry or/and exit costs: Recent research in international economics, employing theoretical analysis and assessment of firm level data clearly confirms that “sunk costs matter” (Godart, Goerg and Goerlich 2009). Setting up of global export networks coincides with substantial set up costs which to a large extent can not be recouped once a firm leaves the export market or terminates its international customer-supplier relationships. Examples of sunk costs of entering export markets are those of information gathering on the new market (costs for market research), setting up distribution and service networks, bearing the costs of establishing a brand name through advertising, and bringing the foreign product into conformity with domestic health regulations, etc. These costs are firm-specific and cannot be resold on exiting the market, at least in terms of their total value, being therefore regarded as (partially) irreversible investments (Kannebley 2008, Roberts and Tybout 1997). The literature on German firm export decisions has found considerable persistence in export status over time (Bernard and Wagner, 2001).

Based on the arguments above, a non-linear reaction of exports to exchange rate changes seems reasonable: Small exchange rate changes will only have weak effects, however stronger exchange rate changes with an monotonously ongoing trend into one direction, will at some point (let it be named “*pain threshold*”) result in larger reactions of the export volume. The exchange rate which forces the firm to a change of the volume of its export activity (i.e. the pain threshold) will be highly product dependent and will differ widely from company to company and from sector to sector (von Wartenberg, 2004). There is *heterogeneity of the exchange rate threshold across firms*, i.e. on the micro level: On the one hand, suppliers of niche products, such as in the field of specialized mechanical engineering or certain segments of the automobile business can perhaps shrug off the increase in value of the euro with comparative ease, while firms with standard products have a huge problem with a strong euro. Moreover, dependent on *past* exchange rate movements, the firms have earlier decided on their export activity status and e.g. spent sunk costs on market entry investments at a time when the exchange rate was favourable – or, vice versa, may have left the export markets if the exchange rate was unfavourable. Thus past decisions are determining the exporters current reaction to exchange rate movements. This type of path-dependence (not only) in foreign trade is associated with the term “hysteresis” (Baldwin, 1989, 1990, and Dixit, 1994).

Empirically addressing the phenomenon of non-linear reactions of exports is not straightforward. Since firms are (due to differences concerning e.g. their pricing-behaviour, their sunk cost structure etc.) heterogeneous concerning their reaction on exchange rate changes, the demanded micro data may not be available. However, aggregation of non-linear path-dependent microeconomic activity to a sectoral or macroeconomic analysis is not straightforward as well, since the path-dependent dynamic pattern may differ between the micro perspective of a firm and the aggregated macro perspective of an entire sector/economy consisting of *heterogeneous* firms (see discussion in Göcke, 2002).

In this contribution we present an approach which captures the path-dependent non-linear dynamics on a macro level called *play-hysteresis*, since it shows an analogy to mechanical play. Play is integrated into a standard regression framework. This has the advantage of a lower demand concerning the underlying data, since macro-data can be used. Furthermore, by developing a theory that is testable using more readily available macro data, the paper brings hysteresis closer to the applicability (e.g. for policy makers).

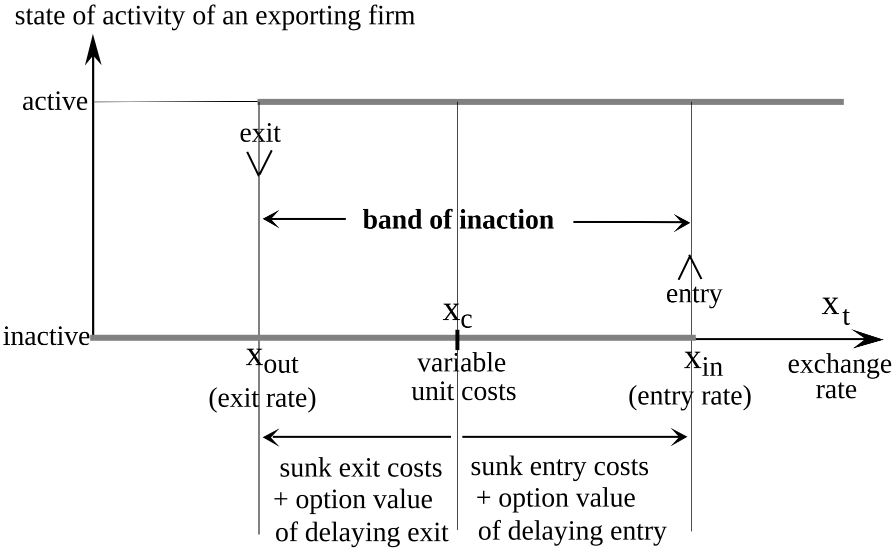
The paper proceeds as follows. In section 2, we present a simple model which serves to capture the non-linear hysteresis-type dynamics inherent in the relation between exchange rate and exports. Taking this model as a starting point, we develop an algorithm describing (macroeconomic) play-hysteresis and implement it into a regression framework in section 3. In section 4, we estimate the exchange rate impacts on German exports to some export destinations outside the €-Zone, differentiating between intervals of weak and strong reaction. Section 5 concludes.

2. Hysteresis in exports: A ‘band of inaction’ from a microeconomic perspective

Hysteresis in foreign trade generally occurs if sunk market-entry costs exist (Baldwin 1989, 1990). Potentially exporting firms must expend market-entry investments, e.g. in setting up a distribution and service network or for introductory sales promotion, in order to sell in the export market. These expenses are firm-specific and cannot be recovered if the firm later wants to leave the market; i.e. the entry costs are sunk. If the prices on the export market do not change in proportion to the exchange rate, the exporting firms have to bear revenue changes in their home currency when the exchange rate alters. If the foreign currency appreciates (i.e. the home currency depreciates), a market entry may become profitable, namely under consideration of the sunk entry costs.

After a firm has entered the export market, the foreign currency may depreciate. However, as long as the variable costs are covered, once in the market, it is still profitable for the firm to sell. A previous entry is not fully reversed due to entry costs which have to be considered as sunk ex post. Analogous effects would result in the case of sunk exit costs. The resulting reaction pattern to exchange rate changes for a single exporting firm is depicted in Figure 2. The exchange rate x is defined as the home currency price of foreign exchange. An exchange rate x_c exactly compensates for the variable unit costs of the firm. A devaluation of the home currency (i.e. an increase of e) increases the unit revenues finally changed back into the exporters home currency. Since the sunk entry costs must be covered, a market entry requires an entry exchange rate x_{in} which exceeds the variable costs (x_c). A previously active firm will exit if the losses are larger than the sunk exit costs. Hence the exit trigger x_{out} must be located below x_c. Seen on the whole, thus, the entry and the exit triggers generally differ in a situation with sunk entry and exit costs. The microeconomic path-dependence occurs discontinuously if entry or exit trigger rates are passed.[2] Combining both triggers results in a ‘band of inaction’. Inside this band, the current exchange rate does not unambiguously determine the current state of the firm’s activity.

Figure 2 – *Discontinuous micro hysteresis loop: export activity of a single firm*



Uncertainty, e.g. about the future exchange rate, reinforces the hysteresis characteristics via option value effects.[3] Since an exit will destroy the market entry investments, an exporting

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[2] According to Krasnosel’skii and Pokrovskii (1989), p. 263, this pattern corresponds to a so-called “non-ideal relay”.

[3] For a comprehensive treatment of uncertainty effects see Dixit, Pindyck (1994). For an empirical application to trade see, based on macro time series, Parsley and Wei (1993). For studies based on micro panel data see Roberts and Tybout (1997) and Campa (2004).

firm may stay when the home currency devalues even if it is currently losing money. If the devaluation would prove to be only transitory, an immediate exit could turn out to be a mistake. Hence, under uncertainty the opportunity of a “wait-and-see”-strategy shifts the exit trigger to the left. Analogously waiting with an entry in a situation with uncertainty shifts the entry trigger to the right. Thus, the “band of inaction” is widened by uncertainty.

Exchange rate changes will result in substantial revenue changes in the home currency if the price elasticity of demand in the export market is high. Vice versa, for a low price elasticity of demand, exchange rate changes do not result in severe unit revenue changes. Thus, the band-of-inaction will be the wider, the lower the demand elasticity is, the higher the absolute values of the sunk entry and exit costs are, and the higher is the uncertainty about the future situation of the exporter.

On a microeconomic level hysteresis occurs via a band of inaction, i.e. differences between both thresholds. Belke and Göcke (2005) focus on the shape and the location of a *macroeconomic* hysteresis loop, i.e. on the *problem of aggregation*.[4] Aggregation is not trivial if heterogeneity regarding the value of sunk exit/entry costs and/or the level of uncertainty about future market situation and/or the elasticity of demand is taken into account, i.e. if the entry and exit trigger rates are different for different exporting firms. In this (realistic) case of heterogeneity the transition from the micro to the macro level leads to a *change of the hysteresis characteristics*: the aggregate hysteresis loop (as known from magnetics) shows no discontinuities. However, a dynamic pattern not very different to a “band of inaction” remains.

Belke and Göcke (2005) show that even the macro behavior can be characterized by areas of weak reactions which are – corresponding to mechanical play – called “play”-areas.[5] Persistent aggregate (export) effects do not result from small changes in the forcing (exchange rate) variables, as far as the changes occur inside a play area. However, if changes go beyond the play area, sudden strong reactions (and persistence effects) of the output variable (i.e. exports) occur.[6] The specific realization of the exchange rate which materializes instantly after the complete passing of the play area can be denoted as a *“pain threshold”*, since,

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[4] For a suitable aggregation procedure from micro to macro hysteresis see Amable et al. (1991), Cross (1994), and Belke and Göcke (2005).

[5] For play hysteresis, see Krasnosel’skii and Pokrovskii (1989), pp. 6 ff. See Göcke (2002) for different types of hysteresis.

[6] For an empirical macro analysis of ‘spurts’ in investment implicitly based on *micro*-threshold models see Darby et al. (1999). See Pindyck (1988), pp. 980 f., Dixit and Pindyck (1994), pp. 15 f., for a non-technical description of ‘spurts’ based on a microeconomic sunk cost mechanism.

passing this realisation of the exchange rate, the reaction of exports to changes in the exchange rate becomes much stronger. However, play-hysteresis is in two aspects different to the micro-loop. First, as mentioned the play-loop shows no discontinuities. Second, analogous to the play in mechanics (e.g. when steering a car) the play area is shifted with the history of the forcing variable (exchange rate): Every change in the direction of the movement of the forcing variable starts with traversing a play area. Only after the play is passed, a spurt reaction will result, if the forcing variable continues to move in the same direction.

In the following section, a straightforward empirical framework to test for a play-type impact of the exchange rate on exports is presented. We use an algorithm developed in Belke and Göcke (2001) describing play-hysteresis and implement it into a regression framework.

3. An empirical model of play-hysteresis

3.1 A linear approximation of exchange rate impacts on exports

In order to convey an impression of the simplified linearized play-dynamics – as theoretically developed by Belke and Göcke (2001, 2005) – we first illustrate the implications based on the interpretation of Figure 3. Here, we assume a constant width p of the play area to simplify issues. We start with an initial situation in point A (x_0) located on the upward leading (right) spurt line, (after changing direction) a decrease in the forcing variable x results in entering the play area. A weak 'play' reaction results until the entire play area p is passed. The downward leading spurt line starts in point G with x_5 (with: $p = x_0 - x_5$). In the play area only a weak reaction of the dependent variable y follows from changes in x. A further decrease of x would induce a strong response of y along the (left) downward leading spurt line.

Alternatively, one may think of an increase in x starting from x_0 (A) up to x_1 (point B) and a subsequent decrease to x_2 (C). The corresponding reaction of y initially evolves along the right spurt line. With an increase along the spurt line from $A \rightarrow B$ the relevant play area is vertically shifted upward from line GA to line DB ($p = x_1 - x_3$). The decrease from x_2 (C) to x_3 (D) again takes place in a play area.[7] This play area is penetrated by an extent 'a' which is explicitly depicted. Consider next a decrease $x_2 \rightarrow x_3 \rightarrow x_4$ ($C \rightarrow D \rightarrow E$). After having passed the entire play width p in point D (x_3), a strong reaction on the downward leading (left) spurt line up to point E results. In this situation, a decrease (i.e. a devaluation of the foreign

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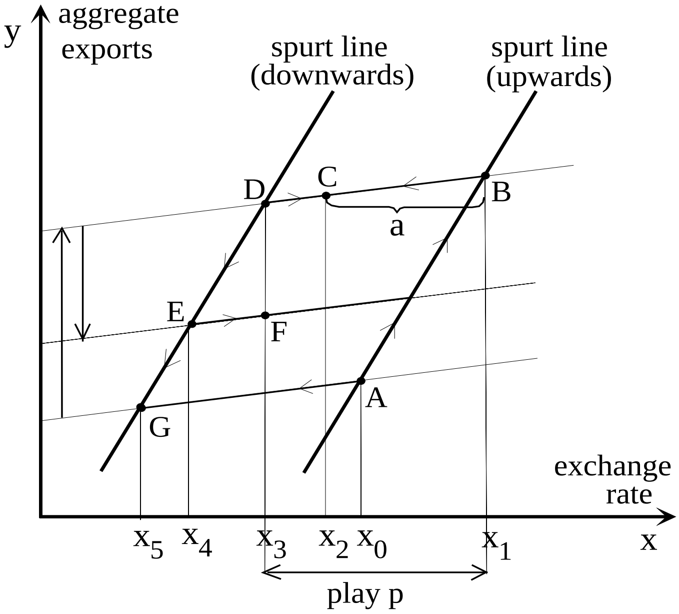
[7] In the case of 'mechanical play' there even would not be any reaction of y inside the play area. See Krasnosel'skii and Pokrovskii (1989), p. 8.

currency) suddenly leads to a strong decrease of the exports. Thus, x_3 is a kind of “pain threshold”. However, this “pain threshold” is not a constant trigger level as in the micro loop, but path-dependent, since the play lines are vertically shifted by movements along the spurt lines. The play area is shifted in the opposite direction as before, so that for a subsequent increase again to x_3 (F) the reaction is described by line EF.

Figure 3 – *Linear play-hysteresis and spurt areas*



3.2 An algorithm capturing linear play

In the following, we present a version of a play algorithm which was originally developed by Belke and Göcke (2001, 2005) for the analysis of employment hysteresis and finally adapt it to our main research question, i.e. the identification of an exchange rate “pain threshold” for German exports. The change in the forcing variable x (Δx) may occur either inside the play area p inducing a weak reaction or on a spurt line resulting in a strong reaction of the dependent variable y (Δy). The movement of x inside the play area is Δa (and cumulated as a) and analogously the movement in the spurt area is Δs. We start with a special case, when Δx enters a play area. Let this change be denoted as Δx_j^s. According to Figure 3 this corresponds to the trajectory B→C→E. In the past the movement of x has led to j changes between the left and the right spurt line. The new change Δx_j^s may enter the play area to an extent of Δa_j or even pass the entire play p and enter the opposite spurt line by the fraction Δs_j. Due to starting from a spurt line the cumulated movement inside the play area a_j equals the change Δa_j. The trajectory B→C in Figure 3 might serve as an illustration of the distance "a". These considerations are usefully summarized by the formal expression:

$$(1) \qquad \Delta x_j^S = a_j + \Delta s_j \qquad \text{with: } \Delta s_j = \begin{cases} \operatorname{sign}(\Delta x_j^S) \cdot (|\Delta x_j^S| - p) & \text{if } (|\Delta x_j^S| - p) > 0 \\ 0 & \text{else} \end{cases}$$

The change in the independent variable y (Δy) induced by Δx_j^S is composed of the weak play reaction (B→C) and – by occasion – additionally of a strong spurt reaction (D→E). Let the parameter α denote the weak play reaction and $(\alpha + \beta)$ the strong spurt reaction:

$$(2) \qquad \Delta y_j^S = \alpha \cdot a_j + (\alpha + \beta) \cdot \Delta s_j \qquad \text{with: } |\alpha| < |\alpha + \beta|$$

The play line is shifted vertically by spurt movements. The cumulated vertical displacement V_{j-1} of the relevant play line as a result of all previous movements on both spurt lines is:

$$(3) \qquad V_{j-1} = \beta \cdot \left[\sum_{i=0}^{j-1} \Delta s_i \right] = \beta \cdot s_{j-1} \qquad \text{with: } s_{j-1} \equiv \sum_{i=0}^{j-1} \Delta s_i$$

The dependent variable is determined by the shift V induced by past spurts and the current reaction Δy_j^S:

$$(4) \qquad y_j = C^* + V_{j-1} + \Delta y_j^S = C^* + \beta \cdot \sum_{i=0}^{j-1} \Delta s_i + \alpha \cdot a_j + (\alpha + \beta) \cdot \Delta s_j$$

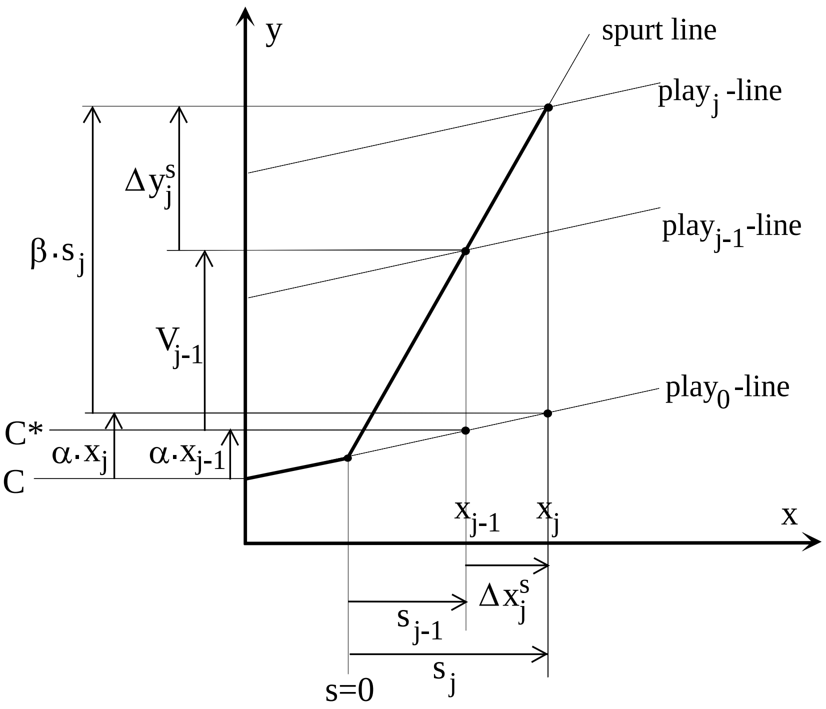
$$\Rightarrow \qquad y_j = C^* + \beta \cdot \sum_{i=0}^{j} \Delta s_i + \alpha \cdot \Delta x_j^S$$

$$\Rightarrow \qquad y_j = C^* - \alpha \cdot \sum_{i=0}^{j-1} \Delta x_i + \beta \cdot \sum_{i=0}^{j} \Delta s_i + \alpha \cdot \Big(\sum_{i=0}^{j-1} \Delta x_i + \Delta x_j^S \Big) \qquad \text{with: } C \equiv C^* - \alpha \cdot \sum_{i=0}^{j-1} \Delta x_i$$

$$\Rightarrow \qquad y_j = C + \alpha \cdot x_j + \beta \cdot s_j$$

Figure 4 conveys an impression of the transformations of equation (4). As a result, the play hysteresis loop is captured by a simple linear equation based on an artificial variable s_j. This "*spurt variable*" s_j summarizes all preceding and present spurt movements leading to a shift of the current relation between x and y.

Figure 4 – *Shift of the play-lines by past spurts and the current reaction* Δy_j^s



Of course, an accumulation by means of an index j describing the past changes between the spurt lines can be substituted by an accumulation over an explicit time index t. Additional non-hysteretic regressors (e.g. z_t) may be included to arrive at a suitably generalized presentation of the hysteretic process:[8]

$$(5) \qquad y_t = C^* + \beta \cdot \sum_{k=0}^{t} \Delta s_t + \alpha \cdot \Delta x_t + \lambda \cdot z_t$$

$$\Rightarrow \qquad y_t = C + \alpha \cdot x_t + \beta \cdot s_t + \lambda \cdot z_t.$$

4. Empirical analysis

4.1 Existing studies

The hypothesis of hysteresis in foreign trade was initially tested by Baldwin (1990) and Krugman and Baldwin (1987) based on macroeconomic time series for the U.S. economy by employing dummy variables associated with periods of exchange rate appreciation. Parsley and Wei (1993) came up with empirical models that try to capture the asymmetric effect of real exchange rate fluctuations and real exchange rate volatility on the imported quantities.

However, they cast doubt on the validity of the hysteresis hypothesis as an explanation of the persistent U.S. trade deficits in the 1980s. Based on micro firm level data, and thus with a focus on the discontinuous micro-hysteresis (however, emphasizing the heterogeneity of firms) Roberts and Tybout (1997) and Campa (2004) discovered sunk cost hysteresis to be an important factor in determining export market participation. Agur (2003) has found empirical evidence of structural breaks in the exchange rate import volume relation as a consequence of exchange rate extrema. Using a threshold cointegration model of Brazilian sectoral foreign trade data, Kannebley (2008) was able to identify an asymmetric (i.e. hysteretic) adjustment in 9 of 16 sectors.

Compared to existing studies of hysteresis in foreign trade, our approach is closer to the original concept of a *macroeconomic* “hysteresis *loop*”, since (i) it is not based on the discontinuous non-ideal relay interpretation as in the microeconomic firm level case and since (ii) the path-dependent structural breaks in the macroeconomic relations are not added to the system as an exogenous information. On the contrary, in our approach the structural shifts are explicitly determined by the history of the exchange rate, and the (path dependent) relation of exports to the exchange rate is simultaneously estimated.

4.2 Characteristics of the regression model and the hypothesis for testing play effects

The ‘play regression’ model displays the following characteristics: It is based on linear segments, where adjacent sections are linked (by so called ‘knots’, in Figure 3 these knots are e.g. points B, D, E in the case of the input path $x_1 \rightarrow x_3 \rightarrow x_4$.). The position of the linear partial function and the transition between the sections is determined by the past path of an input variable x. The model is a special case of a switching regression setting, since adjacent sections are joined.[9] The positions of the knots are a-priori unknown and depend on the magnitude of the play area p, which has to be estimated. The knots divide the relation between x and y into sections with two different slopes (for $\beta \neq 0$). The number of parameters describing the complex dynamics is low: only the basic slope α, the slope difference β and the play width p are to be determined.

We suppose that the standard regression model assumptions hold: the error term is independently, identically and normally distributed with a constant finite variance over all

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[8] For a detailed description of the algorithm calculating the artificial spurt variable s_t and for the implementation into batch programs within standard econometric software packages see Belke and Göcke (2001) and the appendix.

[9] For linear spline functions and linear switching regressions see Poirier (1976), p. 9 and p. 117.

sections, and the regressors are measured without any error and are not correlated with the error term.

Our model is non-linear in its parameters, since the knots are not known a-priori and since the play width p has to be estimated in order to determine the spurt variable s. The assumptions regarding the error term and the regressors guarantee that the OLS-estimators are best linear unbiased estimators (BLUE) in a standard regression model and allow the OLS-estimator to be regarded as a maximum likelihood estimator. If the knots are a-priori unknown, discontinuities and local maxima in the likelihood function result. However, if the adjacent sections are joined in a switching regression models the OLS-/ML-estimator leads to consistent and asymptotically normally distributed estimates.

Unfortunately, the finite sample properties of the play regression model remain problematic: The parameter estimates are not even approximately normally distributed for small samples and local maxima in the likelihood function may occur.[10] Moreover, the standard regression model assumptions may not be fulfilled. For example non-stationary variables might imply non-finite variances. Furthermore, the play dynamic represents a mixture of the short-term and the long-term dynamics, which obstructs the application of standard cointegration analysis. Unfortunately, we are not aware of any technique which is directly applicable to our specific model and therefore delivers the distribution and the critical values of the estimators. Thus, any solution to these problems is clearly beyond the scope of this paper.

In order to minimize the residual sum of square a grid search over the width of a time invariant play width parameter $p_t = p = \gamma$ is executed (with a constant width as the most simple case of play dynamics). For every given point of the γ-grid the algorithm “recognizes” the switches, and for the given γ the spurt variable s is computed from the actual input (exchange rate) series. The size of γ is predetermined for each grid point. Then the respective OLS-estimation of α and β for each grid point is straightforward since s enters the regression equation (5) in a linear way. The final OLS-estimate of the play parameter is found by the γ-grid-value with the maximum R-squared (i.e. the minimum of the residual sum of squares).

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[10] See Hujer (1986), pp. 231 ff., Poirier (1976), pp. 108 ff., pp. 117 ff. and p. 129, Hudson (1966) and Hinkley (1969) for the small sample properties of the ML- (OLS-) estimates in a (spline) model with unknown but continuous switches.

The relevant hypotheses to be tested must refer to the equations:[11]

(5') $y_t = C + \alpha \cdot x_t + \beta \cdot s_t(\gamma) + \lambda \cdot z_t$ with: $|\alpha| < |\alpha + \beta|$

(6) $p_t = \gamma$ with: $\gamma \geq 0$.

Assessing the relevance of play, we have to test the hypothesis (H1) $\beta \neq 0$ against the alternative $\beta = 0$.[12] If one (for the moment) neglects possible limitations on inference resulting from, for instance, non-finite variances of the variables, the OLS-estimates of the respective equations can be regarded as asymptotically unbiased (i.e. consistent) and asymptotically normally distributed. However, since the small sample properties remain problematic we refrain from further conclusions concerning exact inference and for the moment only convey a broad-brush view of the basic pattern of the results. Therefore, the following regression results serve more as a *first illustration of the functioning of our regression algorithm* and the main direction of results rather than a basis for exact inference.

4.3 Estimating play-effects in German exports as an example

In order to check for the empirical relevance of the hysteresis model for German exports, we now estimate equation (5) which generalizes hysteretic behavior of exports dependent on movements in the exchange rate. In our empirical application, we use export data for some of the most important German export destinations outside the euro area – namely Denmark, Japan, Norway, Switzerland and the United States[13] – as the dependent variable, disaggregated by product groups (SITC), and the national currency-to-€-exchange rate as the hysteretic input variable. To be as parsimonious as possible, we employ foreign real GDP, a linear trend and seasonal dummies as additional non-hysteretic explanatory/controlling variables.

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[11] However, generalizing the model in a way where the play width p_t is not constant and determined by other variables is possible (Belke and Göcke, 2005). For instance, the higher an uncertainty variable u_t is, the more important are option value effects of waiting, and thus the play area is expected to widen. See eq. (12) in the appendix for this generalization.

[12] According to Belke and Göcke (2001, 2005), the hypothesis to be tested might even be more restrictive, since in terms of absolute numbers a weaker play and a stronger spurt reaction are assumed as the "typical" hysteresis pattern (i.e. $|\alpha| < |\alpha + \beta|$)

[13] Our final country selection is predominantly oriented at data availability and the specific kind of prevailing exchange rate regime. Our overall aim was to arrive at a homogeneous data set. China was excluded because of its governmentally directed exchange rate. During the observation period, Russia as well as Turkey went through currency reforms that cannot easily be accounted for in our algorithm. The Eastern exporting nations were also excluded because exports to these countries are assumed to rather depend on other effects such as transition and catching up processes. India, Korea, Brazil and Australia were ignored due to data limitations. Exports to the UK do not reveal any evidence in favour of hysteresis effects. Hence, we omitted them ex post.

The exact definitions of the time series are used are as follows. Nominal exports are denoted in current € and taken from the Eurostat database. The export series are deflated by the German export price deflator. Exchange rates are (monthly) averages as documented by the Deutsche Bundesbank,[14] and real exchange rates are calculated using the price deflator of German exports divided by the price deflator of domestic demand of the export destination. The deflators and real GDP time series are from the Eurostat database. Our estimation period ranges from 1995Q1 to 2010Q3.

As an example we start with a standard regression of German exports to the US of SICT 78 goods (Road vehicles including air-cushion vehicles) on the price adjusted real $/€ exchange rate (RER), the US-GDP and, additionally, a linear trend plus dummy variables for the first 3 quarters (Q1 to Q3). As a first stage we exclude play or spurt effects (i.e. applying the restriction $\beta = 0$). The corresponding results are stated in Table 1. The estimated coefficients of all regressors are (according to the t-statistics) highly significant and display the theoretically expected sign. The US GDP variable enters with a lag of one quarter. Lagged GDP data are used because they produce the best fit in the following regressions and avoid problems of reverse causation.[15] In contrast, the real $/€ exchange rate enters contemporaneously. Otherwise, J-curve-effects might occur which might severely interfere with the hysteretic dynamics sub-system. We employ this general setting in all following estimations.[16] Our sample period consists of 15 years (1995Q1-2010Q3), which is for the $/€ rate representing just "one complete cycle", where starting from a high external value of the euro, at first a trendwise euro depreciation and then an ongoing appreciation trend are manifesting themselves. Of course, data would be available for a longer period. However, we did not exploit them because we intend to avoid possible structural break related to German reunification which would potentially interfere with structural shifts caused by play dynamics.

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[14] In fact, we employ synthetic euro exchange rates, which consist of hypothetical euro exchange rates before 1999 and the monthly average time series of ECB euro reference exchange rates since 1999. The hypothetical euro exchange rate before 1999 is simply calculated based on the DM exchange rates and the fixed DM/€-rate.

[15] Using lagged GDP avoids problems potentially related to endogeneity effects of the dependent variable (exports) and the regressor (GDP). However, we are not able to completely exclude these kinds of effects since export numbers could theoretically contemporaneously affect the exchange rates. But since the exchange rate is the base of our play-dynamics, we are not able to overcome this problem in an easy way (e.g. via using instrumental variables), and must leave this problem for further research.

[16] Our regression is only directed at bilateral effects between two countries and their bilateral exchange rate. Of course, if exchange rate changes differ between export destinations, an exporter could react with substituting/redirecting exports away from the depreciating country towards a third country market. These cross-country effects are not considered. However, from a sunk cost point of view, even redirecting export flows may cause sunk costs, and thus, may show some kind of cross-exchange rate play (with only weak reaction until the country structure of exchange rates changes severely).

Table 1 – *Standard LS regression without play (restriction $\beta = 0$)*

Dependent Variable: German Exports to US (SITC 78)
Sample: 1995Q1 2010Q3
Included observations: 63

	Coefficient	Std. Error	t-Statistic	Prob.
C	-12058.68	2332.604	-5.169622	0.0000
RER	-2338.159	527.2402	-4.434714	0.0000
US-GDP(-1)	0.009290	0.001061	8.751995	0.0000
TREND	-129.8196	18.70930	-6.938775	0.0000
Q1	-367.3165	194.7426	-1.886164	0.0645
Q2	-431.6854	194.9541	-2.214292	0.0309
Q3	-599.8171	194.9287	-3.077110	0.0032

R-squared	0.840447	Mean dependent var	3915.985
Adjusted R-squared	0.823352	S.D. dependent var	1288.790
S.E. of regression	541.6716	Akaike info criterion	15.53164
Sum squared resid	16430857	Schwarz criterion	15.76976
Log likelihood	-482.2465	F-statistic	49.16356
Durbin-Watson stat	0.595938	Prob(F-statistic)	0.000000

As a second step, we estimate γ for a simple case with constant play. In Figure 5 we display a plot of the grid search on different values of γ: The R^2 sequence shows an absolute maximum at $\gamma = 0.23$ (with $R^2 = 0.905753$). The R^2 minimum at $\gamma = 0$ ($R^2 = 0.840447$) exactly corresponds to the linear standard model stated in Table 1. The estimation results of the spurt/play regression with an artificial spurt-variable (SPURT) based on the constant play-width $p = \gamma = 0.23$ is presented in Table 2.

Figure 5 – *R^2 resulting from a one-dimensional grid search over constant play γ*

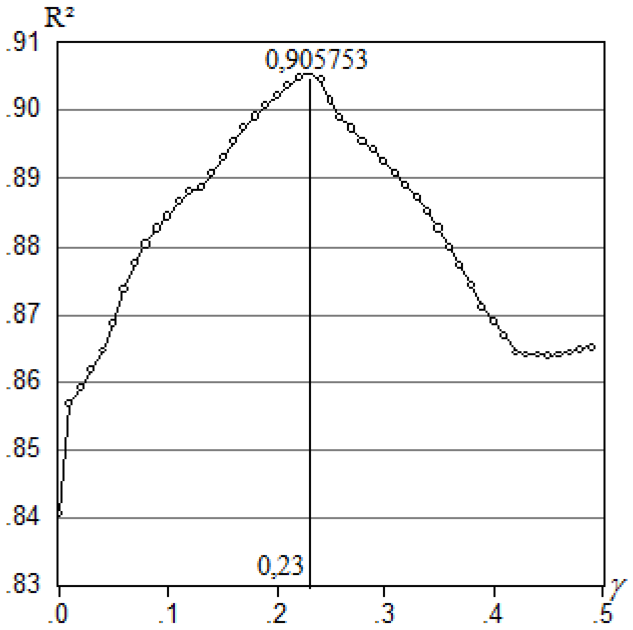


Table 2 – *LS regression with constant play* $p = \gamma = 0.23$

Dependent Variable: German Exports to US (SITC 78)
Sample: 1995Q1 2010Q3
Included observations: 63

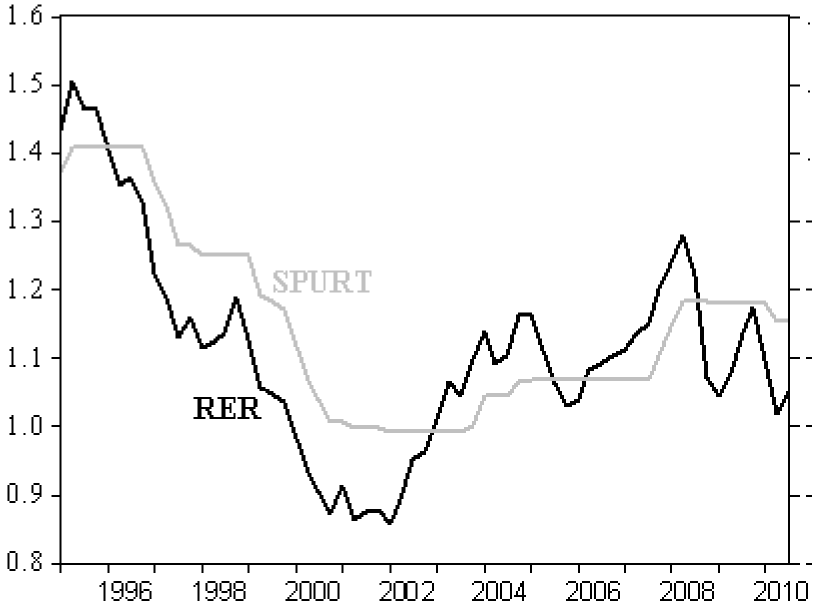
	Coefficient	Std. Error	t-Statistic	Prob.
C	-8505.976	1898.322	-4.480788	0.0000
RER	907.2109	665.9987	1.362181	0.1787
SPURT	-6547.481	1060.601	-6.173368	0.0000
US_GDP(-1)	0.004980	0.001079	4.612946	0.0000
TREND	-70.12511	17.43641	-4.021763	0.0002
Q1	-336.7113	151.1087	-2.228272	0.0300
Q2	-422.7756	151.1983	-2.796166	0.0071
Q3	-596.3865	151.1728	-3.945066	0.0002

R-squared	0.905753	Mean dependent var	3915.985
Adjusted R-squared	0.893758	S.D. dependent var	1288.790
S.E. of regression	420.0789	Akaike info criterion	15.03693
Sum squared resid	9705645.	Schwarz criterion	15.30907
Log likelihood	-465.6633	F-statistic	75.51031
Durbin-Watson stat	0.963524	Prob(F-statistic)	0.000000

Again, all coefficients display the theoretically expected signs. With respect to the hypothesis (H1) $\beta \neq 0$ the estimated coefficient of the spurt variable is $\beta = -6547.481$ with an empirical t-value of –6.17. Note that, as expected, the spurt-variable substitutes the effects of the original real $/€ exchange rate, which in the linear standard regression in Table 1 was $\alpha = -2338.159$ (t=–4.43), and now in the play-regression vanishes to an insignificant effect ($\alpha = 907.2109$, $t = 1.36$). Furthermore, the absolute effect of spurt in the play-regression is stronger, compared to the original exchange rate effect in the linear regression. However, since the small sample properties of our regression model are unknown, the t-values are most probably not student-t-distributed. Nevertheless, this high empirical t-realization (which is about three times as high as the 5% critical value in case of a standard student-t-distribution) represents at least a strong hint at the relevance of hysteretic play.

Finally, Figure 6 conveys a graphical impression of the time sequence of the original real $/€ exchange rate (RER, left scale) and of the respective SPURT (right scale) which captures the strong impact of exchange rate changes after passing the play area (i.e. after passing a kind of "pain threshold"). The time path of the spurt variable shows of course similarities to the original real exchange rate path. However, limited variability of the original real exchange rate series inside the play area (of width $\gamma = 0.23$) is filtered away and periods of inaction emerge, exhibiting no variation of the spurt variable due to play/inaction effects. Only large/monotonous changes in the real exchange rate are reflected by the artificial spurt series.

Figure 6 – *Real exchange rate and the resulting spurt variable (γ =0.23)*



If the spurt variable changes, this simultaneously shifts the current position and the borders of the play area. The up to now most recent shift of the play position/borders corresponds to the exchange rate extremum of spring/summer 2010. Thus, the corresponding upper bound exchange rate of the play area was valid till the end of our estimation sample, which is the third quarter 2010. The corresponding upper bound of the play area of the real exchange rate can be calculated taking the 2010 spurt value and adding the identified play γ. After correction for the deflator effects, this estimated upper bound of the play area in terms of the nominal exchange rate is about *1.55 US-$ per €.* This exchange rate threshold induces, once it is passed by a further $-depreciation (or €-appreciation), a strong spurt reaction of German exports, and thus can be interpreted as a kind of *"pain threshold"* for the period from 2010 up to now.

Exports to different export destinations – evidence on a sectoral level

Further play-regressions were (for the same sample period) calculated for 6 different product groups of German exports and for 5 different export-receiving non-euro countries (Denmark, Japan, Norway, Switzerland and the United States). Table 3 summarizes these results.

Table 3 – O*verview of the regression results with constant play for different countries receiving German exports and different sectors*

		SICT Group					
		0 to 4	5	6	78	7 without 78	8
Destination of German exports	Denmark	$\alpha = 171$ *** $\gamma = 1.55$ $\beta = -254$ t= –7.89 ***	$\alpha = 44.9$ $\gamma = 1.5$ $\beta = -442$ t= –8.49 ***	$\alpha = 262$ *** $(\alpha+\beta > 0)$ $\gamma = 1.2$ $\beta = -248$ t= –5.46 ***	$\alpha = 61.2$ $\gamma = 1.15$ $\beta = -219$ t= –4.23 ***	$\alpha = 183$ *** $\gamma = 1.5$ $\beta = 608 > \mathbf{0}$ t= 5.40 ***	$\alpha = 50.1$ ** $\gamma = 1.15$ $\beta = -77.9$ t= –4.45 ***
	Japan	$\alpha = 0.01$ $\gamma = 60$ ***(bi.)*** $\beta = 2.20 > \mathbf{0}$ t= 8.87 ***	$\alpha = -0.92$ $\gamma = 13.5$ $\beta = -2.55$ t= –2.27 **	$\alpha = -0.34$ * $\gamma = 44$ $\beta = -1.33$ t= –5.74 ***	$\alpha = 0.96$ $\gamma = 22$ $\beta = -7.00$ t= –2.94 ***	$\alpha = -0.86$ $\gamma = 44$ $\beta = -5.94$ t= –9.26***	$\alpha = 0.21$ $\gamma = 50$ $\beta = -3.86$ t= –9.44 ***
	Norway	$\alpha = 7.28$ $\gamma = 3.2$ $\beta = -36.2$ t=–3.45 ***	$\alpha = 7.94$ * $\gamma = 3.2$ $\beta = -98.8$ t=–10.6 ***	$\alpha = 64.0$ *** $\gamma = 2.7$ $\beta = -106$ t= –3.82 ***	$\alpha = -21.3$ $\gamma = 2{,}95$ $\beta = -204$ t= –3.69 ***	$\alpha = 58.5$ * $\gamma = 3.25$ $\beta = -97.1$ t= –1.50	$\alpha = 15.2$ *** $\gamma = 2.9$ $\beta = -30.3$ t= –2.96 ***
	Switzerland	$\alpha = 1689$*** $\gamma = 0.34$ $\beta = -2797$ t= –7.78 ***	$\alpha = -384$ $\gamma = 0.41$ $\beta = -800$ t= –1.79 *	$\alpha = 1018$*** $\gamma = 0.27$ $\beta = -1620$ t= –4.01 ***	$\alpha = -550$* $\gamma = 0.09$ $\beta = -582$ t= –1.25	$\alpha = 1389$* $\gamma = 0.069$ $\beta = -2146$ t= –1.90 *	$\alpha = -74.7$ $\gamma = 0.43$ $\beta = -879$ t= –2.53 **
	United States	$\alpha = 204$ $\gamma = 0.13$ $\beta = 288 > \mathbf{0}$ t= 1.28	$\alpha = -925$ ** $\gamma = 0.31$ $\beta = 1375 > \mathbf{0}$ t= 2.51 **	$\alpha = -482$ * $\gamma = 0.13$ $\beta = 602 > \mathbf{0}$ t=1.91 *	$\alpha = 907$ $\gamma = 0.23$ $\beta = -6547$ t= –6.17 ***	$\alpha = 2601$*** $\gamma = 0.44$ ***(bi.)*** $\beta = -3103$ t= –2.82 ***	$\alpha = 208$*** $\gamma = 0.3$ $\beta = -318$ t= –2.10 **

α: estimated coefficient for the original real exchange rate (RER)

β: estimated coefficient for the spurt exchange rate variable (SPURT)

γ: estimated play width

level of significance (student-t statistic): *** for 1 % , ** for 5 % , * for 10%

(bi.): estim. SPURT variable turned out to be a binary variable capturing a one-time structural break

The first five product groups (SITC 0 to 4) are combined and pooled, as they represent the classic primary sector.[17] Product group 9 – i.e. commodities and transactions not classified elsewhere in the SITC – was skipped because of unknown real compositions of products in this group. Group 7, which makes up for 46.8 percent of all exports for the year 2009 (46.4 percent of the exports to the countries analyzed in this paper), was split into sub-group 78 (Road vehicles – including air-cushion vehicles) and “group-7-without-subgroup-78” in order

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[17] SITC Group 0 to 4 includes goods such as food, live animals, beverages, tobacco, crude materials, inedible, mineral fuels, lubricants and related materials, animal and vegetable oils, fats and waxes. For more detailed definitions please refer for example to the UNSTAT website. These groups of products were also combined because they account for only about 9.9% of German exports for 2009 and for about only 7.6% of all exports to the included countries. Own calculations based on Eurostat data.

to capture the specific importance of the automobile sector. SICT-sub-group 78 accounts for 17.3% of exports to the included countries.[18]

The real exchange rates were for the regressions defined in a way that a “normal” reaction of the exports to the spurt of the exchange rates is expected with a negative coefficient (i.e. if an €-appreciation reduces German exports). A “typical” result of hysteretic play dynamics – as theoretically expected – would be a significantly negative effect of the spurt variable (i.e. $\beta < 0$) and a weaker (or even insignificant) effect of the original exchange rate. For the 30 regressions, the spurt variable showed the “wrong sign” ($\beta > 0$) in 5 cases, and in 1 case the original exchange rate effect was stronger than the estimated spurt effect ($\alpha+\beta > 0$). Regressions with a theoretically unexpected sign are in Table 3 marked by grey shading. For very large sizes of the play width, the computed SPURT variable reduces to a time series with a “┐└”- or “┘┌”-shape time-plot. In these cases SPURT actually reduces to a kind of binary/dummy variable which only captures a one-time shift. Thus, in this limit cases play-hysteretic shift could not be separated from one-time structural breaks.[19] As marked in Table 3, this limit case actually is valid in 2 cases (one overlapping with a “wrong” sign). It has to be noted that the respective t-value of the spurt variable is stated with each entry in the table. On two occasions, the spurt variable shows the expected sign, but is not significant due to low t-statistics. Summarizing, in 21 of 30 cases, the export regressions are in line with “typical” play-dynamics and are leading to “significant” t-statistics for the spurt variable (however, with the mentioned caveats concerning the distribution of the estimators).

5. Conclusions

The paper deals with the impact of the exchange rate on the relationship between German exports and its main determinants. Our aim was to identify a band of inaction for German exports. We rely on a non-linear path-dependent model in which suddenly strong spurts of exports occur when changes of the exchange rate go beyond a so called ‘play area’ (which is similar to the phenotype of play in mechanics). We capture this non-linear dynamics in a simplified linearized way and implement an algorithm describing play hysteresis into a regression framework. For several sub-groups of German total exports our non-linear model including play-hysteresis shows a significant effect of the non-linear play-dynamics. Analyzing some of the largest export partners outside the Euro-zone into which 14.8% of the

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[18] Own calculations based on Eurostat data. The shares are an average for the year 2009.

total German exports were directed, we find hysteretic play-effects in more than 8% of total German exports.

To conclude, the existence of ‘bands of inaction’ (called ‘play’) in German exports should lead to a more objective discussion of peaks in the euro exchange rates in political debates. Not every increase or decrease of the exchange rate will, automatically, lead to positive or negative reactions of the volume of exports. However, a large appreciation of the euro means passing the border of a play/inaction-area (which can be seen as a kind of “pain-threshold”) and results in a strong reaction of exports. Moreover, we show that the play/inaction area is path-dependent – and changes its position with extreme exchange rate movements. Thus, a unique “pain threshold”, for instance, of the $/€ exchange rate does not exist.

References

Agur, I. 2003. Trade-volume Hysteresis - an Investigation Using Aggregate Data. Research Memorandum WO no. 740, De Nederlandsche Bank, Amsterdam, August.

Amable, B., Henry, J., Lordon, F. and Topol, R. 1991. Strong Hysteresis: An Application to Foreign Trade. OFCE Working Paper / Document de travail no. 9103, Observatoire Français des Conjonctures Economiques, Paris.

Baldwin, R. 1989. Sunk-Cost Hysteresis. NBER Working Paper, no. 2911, National Bureau of Economic Research, Cambridge/MA.

Baldwin, R. 1990. Some Empirical Evidence on Hysteresis in Aggregate U.S. Import Prices. in: Gerlach, St., Petri, P.A. (eds.), The Economics of the Dollar Cycle, Cambridge/MA, pp. 235-268.

Belke, A., Göcke, M. 2001. Exchange Rate Uncertainty and Employment: An Algorithm Describing ‘Play’. Applied Stochastic Models in Business and Industry, Vol. 17(2), pp. 181-204.

Belke, A., Göcke, M. 2005. Real Options Effects on Employment: Does Exchange Rate Uncertainty Matter for Aggregation?. German Economic Review, Vol. 62, pp. 185-203.

Bernard, A.B., Wagner, J. 2001. Export Entry and Exit by German Firms. Weltwirtschaftliches Archiv, Vol. 137(1), pp. 105-123.

Campa, J. M. 2004. Exchange Rates and Trade: How Important is Hysteresis in Trade?. European Economic Review, Vol. 48(3), pp. 527-548.

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[19] As an example of this limit case, see group “7-without-78” in the German exports to the US.

Cross, R. 1994. The Macroeconomic Consequences of Discontinous Adjustment: Selective Memory of Non-Dominated Extrema. Scottish Journal of Political Economy, Vol. 41, pp. 212-221.

Darby, J., Hughes Hallet, A., Ireland, J. and Piscitelli, L. 1999. The Impact of Exchange Rate Uncertainty on the Level of Investment. Economic Journal, Vol. 109, C55-C67.

Deutsche Bundesbank 2008. Macroeconomic Effects of Changes in Real Exchange Rates. Monthly Report, Frankfurt/Main, March, pp. 33-46.

Dixit, A., Pindyck, R.S. 1994. Investment under Uncertainty. Princeton, NY.

Dow Jones International News 2007. Business Lobby Says Euro Has Reached “Pain Threshold”. October 3.

Godart, O., Goerg, H., Goerlich, D. 2009. Back to Normal – The Future of Global Production Networks after the Crisis. Kiel Policy Brief, No. 9, Kiel Institute for the World Economy, September.

Göcke, M. 2002. Various Concepts of Hysteresis Applied in Economics. Journal of Economic Surveys, Vol.16, n.2, pp.167-88.

Hinkley, D.V. 1969. Inference about the Intersection in Two-Phase Regression. Biometrica, Vol. 56, pp. 495-504.

Hudson, D.J. 1966. Fitting Segmented Curves Whose Join Points Have to Be Estimated. Journal of the American Statistical Association, Vol. 61, pp. 1097-1125.

Hujer, R. 1986. Ökonometrische ‘Switch’-Modelle: Methodische Ansätze und empirische Analysen. Jahrbücher für Nationalökonomie und Statistik, Vol. 201/3, pp. 229-256.

Kannebley, Jr. S. 2008: Tests for the Hysteresis Hypothesis in Brazilian Industrialized Exports: A Threshold Cointegration Analysis. Economic Modelling, Vol. 25, pp. 171–190.

Krasnosel’skii, M.A., Pokrovskii, A.V. 1989. Systems with Hysteresis. Berlin.

Krugman, P. Baldwin, R. 1987. The Persistence of the US Trade Deficit. Brookings Papers on Economic Activity, Vol. 1, pp. 1-43.

Parsley, D.C. and Wei, S.-J. 1993. Insignificant and Inconsequential Hysteresis: the Case of U.S. Bilateral Trade. Review of Economics and Statistics, Vol. 75, pp. 606-613.

Pindyck, R.S. 1988. Irreversible Investment, Capacity Choice and Uncertainty. American Economic Review, Vol. 78, pp. 969-985.

Poirier, D.J. 1976. The Econometrics of Structural Change - With Special Emphasis on Spline Functions. Amsterdam.

Roberts, M.J., Tybout, J.R. 1997. The Decision to Export in Colombia: an Empirical Model to Entry with Sunk Costs. American Economic Review, Vol. 87, pp. 545-564.

Stahn, K. 2007. Has the Export Pricing Behavior of German Enterprises Changed? Empirical Evidence from German Sectoral Export Prices. Journal of Economics and Statistics, Vol 227(3), pp. 295 ff.

Strupczewski, J. 2007. UPDATE 1-EU Business Group Says Euro Near Pain Threshold. Reuters, September 17.

von Wartenberg, L. 2004. Quo Vadis U.S. Dollar - Consequences for Transatlantic Trade. Statement, Konrad-Adenauer-Stiftung e.V. (ed.), Washington, May 19.

Annex: An algorithm for calculating the spurt variable

In the following we present a detailed algorithm based on Belke and Göcke (2001) to calculate the extent of the current penetration into the play area a_t and the cumulated spurts s_t. We define four dummy variables describing the current state of the system. For reasons of simplification, some special cases which become relevant if the change in x *exactly* meets the border between play and spurt (e.g. in point D) are not explicitly included below. However, these cases are taken into account in the Eviews version of the algorithm.

A dummy $M_t^{\downarrow}$ indicates a movement starting in a left (downward leading) spurt line. Analogously, $M_t^{\uparrow}$ indicates a start on a right (upward leading) spurt line. Corresponding to Figure 3 e.g. for point E, $M_t^{\downarrow}=1$ holds, and for point B $M_t^{\uparrow}=1$ is valid.

$$(7) \qquad M_t^{\downarrow} = \begin{cases} 1 & \text{if} \quad \Delta s_{t-1} < 0 \\ 1 & \text{if} \quad (\Delta s_{t-1} = 0) \wedge (\Delta x_{t-1} = 0) \wedge (\Delta a_{t-1} = 0) \\ 0 & \text{else} \end{cases}$$

$$M_t^{\uparrow} = \begin{cases} 1 & \text{if} \quad \Delta s_{t-1} > 0 \\ 1 & \text{if} \quad (\Delta s_{t-1} = 0) \wedge (\Delta x_{t-1} = 0) \wedge (\Delta a_{t-1} = 0) \\ 0 & \text{else} \end{cases}$$

Due to the path dependence, information on the current reference spurt line has to transmitted to subsequent periods: The dummies $B_t^{\downarrow}$ and $B_t^{\uparrow}$ indicate the last (and maybe the current) spurt line. In Figure 3 e.g. for point F, $B_t^{\downarrow}=1$ is valid, and $B_t^{\uparrow}=1$ holds for point C.

$$(8) \qquad B_t^{\downarrow} = \begin{cases} 1 & \text{if} \quad \Delta s_{t-1} < 0 \\ 1 & \text{if} \quad (\Delta s_{t-1} = 0) \wedge (B_{t-1}^{\downarrow} = 1) \\ 0 & \text{else} \end{cases}$$

$$B_t^{\uparrow} = \begin{cases} 1 & \text{if} \quad \Delta s_{t-1} > 0 \\ 1 & \text{if} \quad (\Delta s_{t-1} = 0) \wedge (B_{t-1}^{\uparrow} = 1) \\ 0 & \text{else} \end{cases} \qquad \text{with: } B_t^{\uparrow} = 1 - B_t^{\downarrow}$$

Now, we calculate the extent a_t to which the play area p_t is penetrated. We first define an auxiliary variable b_t. Play penetration a_t is calculated based on a comparison of b_t and the play width p_t.

$$(9) \qquad b_t = B_t^{\downarrow} \cdot (1 - M_t^{\downarrow}) \cdot (a_{t-1} + \Delta x_t) + B_t^{\uparrow} \cdot (1 - M_t^{\uparrow}) \cdot (a_{t-1} - \Delta x_t)$$

$$(10)\qquad a_t = \begin{cases} b_t & \text{if}\quad 0 < b_t \le p_t \\ \Delta x_t & \text{if}\quad (M_t^{\downarrow} = 1) \wedge (\Delta x_t > 0) \wedge (\Delta x_t < p_t) \\ -\Delta x_t & \text{if}\quad (M_t^{\uparrow} = 1) \wedge (\Delta x_t < 0) \wedge (-\Delta x_t < p_t) \end{cases}$$

Finally, we define changes in the spurt variable (Δs_t) induced by changes in the input variable (Δx_t):

$$(11)\qquad \Delta s_t = \begin{cases} b_t \cdot [B_t^{\downarrow} \cdot (1 - M_t^{\downarrow}) - B_t^{\uparrow} \cdot (1 - M_t^{\uparrow})] & \text{if}\quad b_t < 0 \\ (b_t - p_t) \cdot [B_t^{\downarrow} \cdot (1 - M_t^{\downarrow}) - B_t^{\uparrow} \cdot (1 - M_t^{\uparrow})] & \text{if}\quad b_t > p_t \\ \Delta x_t & \text{if}\quad [(M_t^{\downarrow} = 1) \wedge (\Delta x_t < 0)] \vee [(M_t^{\uparrow} = 1) \wedge (\Delta x_t > 0)] \\ \Delta x_t - p_t & \text{if}\quad (M_t^{\downarrow} = 1) \wedge (\Delta x_t > p_t) \\ \Delta x_t + p_t & \text{if}\quad (M_t^{\uparrow} = 1) \wedge ((-\Delta x_t) > p_t) \end{cases}$$

The width of the play p_t was not addressed up to now. In a simple case p_t is defined as a constant parameter $p_t=p=\gamma$ witch has to be estimated. However, it is easy to gereralize the model in a way where the play width p_t is determined by other variables. For instance, the higher an uncertainty variable u_t is, the more important are option value effects of waiting, and thus the play area is expected to widen. In technical term this can be expressed in a simple linear way as a function of, e.g., an uncertainty proxy variable u_t:

$$(12)\qquad p_t = \gamma + \delta \cdot u_t \qquad\qquad \text{with:}\; \gamma, \delta \ge 0 \;\text{ and }\; u_t \ge 0 \;\Rightarrow\; p_t \ge 0$$

Table A.1: Implementation of the algorithm into an EVIEWS-batch program

```
SMPL 69.1 98.4

'INPUT AREA
GENR s_up=1      'set 1 for a maximum as an initial extremum (else 0)
!an = 73.3       'first estimation quarter (time of the first extremum in a
spurt area)
!en = 96.1       'last estimation quarter
!n = 24*4+1      'number of sample point (calculated from !an to !en)
!g = 10          'precision of the grid search for the constant play
component
!m = 0           'minimum of the grid search for the constant play component
!b = 20          'maximum of the grid search for the constant play component
!h = 10          'precision of the grid search for the variable play
component
!y = 0           'minimum of the grid search for the variable play component
!v =30           'maximum of the grid search for the variable play component
GENR w =  HYINPUT   'hysteretic input variable
GENR u = UINPUT  'determination of the uncertainty realisation
%ST11= "HYOUTPUT"                                       'dependent variable
%ST12= "C HYINPUT GDP(-1) TREND D1 D2 D3"  'independent variables of the
regression
'END OF INPUT AREA

'INITIALISATION
```

```
SMPL 69.1 98.4
GENR dw=na
GENR d_spurt=na
GENR play=na
GENR spurt=na
GENR bs_do=na
GENR s_do=na
GENR bs_up=na
GENR pb=na
GENR pc=na
GENR pa=na
GENR punkt_do=na
GENR punkt_up=na
GENR dw=w-w(-1)
C=0
matrix(!g,!h) R_2m =0
matrix(!g,!h)  C_11m = 0
matrix(!g,!h)  C_12m = 0
matrix(!g,1) P_CONSTA =0
matrix(1,!h) P_VARIA =0
SMPL !an !an
GENR bs_up=s_up
GENR s_do=1-s_up
GENR bs_do=1-s_up
SMPL !an-1 !an
GENR pa=0
GENR pb=0
GENR pc=0
GENR d_spurt=0
GENR spurt=0
'END OF INITIALISATION

'START OF GRID SEARCH
FOR !0=1 TO !g    'LOOP FOR P_CONSTA
FOR !1=1 TO !h    'LOOP FOR P_VARIA
SMPL !an !en
GENR spurt=0
GENR play = !m+((!0-1)/(!g))*(!b-!m) + (!y+((!1-1)/(!h))*(!v-!y))*u
P_CONSTA(!0,1) = !m+((!0-1)/(!g))*(!b-!m)
P_VARIA(1,!1) = !y+((!1-1)/(!h))*(!v-!y)

IF @MIN(play)>0 THEN

 FOR !2=1 TO !n  'LOOP FOR THE DETERMINATION OF THE SPURT VARIABLE

  SMPL !an+!2 !an+!2

  GENR punkt_do=(pa(-1)=play(-1))*(pa(-1)<>0)*s_up(-1)+(pb(-1)=play(-
1))*(pb(-1)<>0)*bs_up(-1)
  GENR punkt_up=(pa(-1)=play(-1))*(pa(-1)<>0)*s_do(-1)+(pb(-1)=play(-
1))*(pb(-1)<>0)*bs_do(-1)
  GENR s_do=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)<0)+(s_do(-
1)=1)*(d_spurt(-1)=0)*((dw(-1)=0)*(pa(-1)=0))) + punkt_do
  GENR s_up=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)>0)+(s_up(-
1)=1)*(d_spurt(-1)=0)*((dw(-1)=0)*(pa(-1)=0))) + punkt_up
  GENR bs_do=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-
1)<0)+(d_spurt(-1)=0)*(bs_do(-1))) + punkt_do
  GENR bs_up=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-
1)>0)+(d_spurt(-1)=0)*(bs_up(-1))) + punkt_up
  GENR pb=bs_do*(1-s_do)*(pa(-1)+dw) + bs_up*(1-s_up)*(pa(-1)-dw)
  GENR pc=s_do*(dw>0)*dw + s_up*(dw<0)*(-dw)
  GENR pa=pc*(pc<=play) + bs_do*(1-s_do)*(pb>0)*(pb<=play)*pb + bs_up*(1-
s_up)*(pb>0)*(pb<=play)*pb
  GENR d_spurt=s_do*((dw<0)*dw+(dw>play)*(dw-play)) + s_up*((dw>0)*dw+((-
dw)>play)*(dw+play)) + bs_do*(1-s_do)*((pb<0)*pb+(pb>play)*(pb-play)) +
bs_up*(1-s_up)*((pb<0)*(-pb)+(pb>play)*(play-pb))
  GENR spurt=spurt(-1)+d_spurt

 NEXT

ENDIF
```

```
    c=0
    SMPL !an !en
    IF @MEAN(spurt)=0 THEN
    EQUATION eq1.LS %ST11 %ST12
    ELSE
    EQUATION eq1.LS %ST11 spurt  %ST12        'OLS ESTIMATION
    ENDIF

    GENR EC = RESID
    R_2m(!0,!1) = @R2
    C_11m(!0,!1) = c(1)
    C_12m(!0,!1) = c(2)

    c=0
    GENR RESID=na
    GENR EC=na

   NEXT
   NEXT   'END OF GRID SEARCH


   'SEARCH FOR HIGHEST R²

    coef(2) c_und_d
    scalar r2_max=0

    FOR !i=1 TO !g
     FOR !j=1 TO !h
       IF  ( R_2m(!i,!j) > r2_max ) THEN
       r2_max=R_2m(!i,!j)
       c_und_d(1)=p_consta(!i,1)
       c_und_d(2)=p_varia(1,!j)
      ENDIF
     NEXT
    NEXT
```

Transcriptions:

a_t = pa ; $B_t^{\downarrow}$ = bs_do ; $B_t^{\uparrow}$ = bs_up ; b_t = pb ; $M_t^{\downarrow}$ = s_do ; $M_t^{\uparrow}$ = s_up ; p_t = play ; s_t = spurt ; Δs_t = d_spurt ;
u_t = u ; x_t = w ; Δx_t = dw ; y_t = BAI ; γ = c_und_d(1) ; δ = c_und_d(2) .

Comments:

In order to apply the batch program, some information has to be delivered in the `'INPUT AREA`, since the starting point has to be characterized, due to the path dependence of the system. It is necessary to start in a spurt area (with either $M_t^{\uparrow} = \text{s\_up} = 1$ or $M_t^{\downarrow} = \text{s\_do} = 1$). Therefore, the sample has to be truncated on occasion and in the `'INPUT AREA` the variable s_up has to be set to 0 or 1.

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