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Abstract

This paper shows that remunerating required reserves can increase the flexibility of monetary policy. The remuneration at the current repo rate implies constant net marginal interest costs of holding required reserves. This allows the central bank also to change the rate also within a reserve maintenance period without inducing a problematic reserve shifting on behalf of the banks. In the euro area, required reserves are remunerated at an average rate. Therefore, the way in which reserves are remunerated has to be changed in order to make use of the advantage of a higher flexibility of monetary policy.

JEL-Classifikation: E41, E52

Keywords: Monetary policy, monetary policy instruments, required reserves, interbank market, Eurosystem

1 Introduction

In the euro area, credit institutions are required to hold compulsory deposits on the accounts with the Eurosystem[1]. On the holdings of these reserves interest is paid. According to the ECB, this remuneration shall ensure that the minimum reserve system neither puts a burden on the banking system nor hinders the efficient allocation of resources (European Central Bank, 2004b, p. 78). This paper gives a further reason for this remuneration. It shows that paying interest on required reserves enhances the flexibility of monetary policy. However, in order to make use of this advantage, the Eurosystem has to change the way in which it remunerates required reserves.

Currently, in the euro area holdings of required reserves are remunerated at an *average rate*. This rate corresponds to the average, over a reserve maintenance period, of the rates on the Eurosystem's main refinancing operations (MROs).[2] This paper shows that in order to make use of the advantage of a more flexible monetary policy, required reserves have to be remunerated at the *current repo rate*. Table 1 exemplifies these two alternative methods of remunerating required reserves: There is a reserve maintenance period covering four subperiods. The central bank raises the repo rate after two subperiods from 2 % to 3 %. If required reserves are remunerated at the average of the repo rates, they will be remunerated at 2.5 % over the whole maintenance period. If reserves are remunerated at the current repo rate, they will be remunerated at 2 % before and 3 % after the interest rate change.

To see why the remuneration of required reserves at the current rate enhances the flexibility of monetary policy consider the following: If reserves are not remuner-

[1]The term "Eurosystem" describes the institution which is responsible for monetary policy in the euro area, namely the ECB and the national central banks in the euro area. For the sake of simplicity, the terms "ECB" and "Eurosystem" are used interchangeably throughout this paper.

[2]The MROs are credit operations through which the Eurosystem provides the bulk of liquidity to the banking sector in the euro area. They have a maturity of one week and are conducted weekly in the form of a tender procedure. Since a reserve maintenance period lasts over four weeks, required reserves are remunerated at the average of four repo rates. For detailed information on the Eurosystem's MROs and its minimum reserve system see European Central Bank (2006). For details concerning the calculation of the remuneration of holdings of required reserves see also appendix A of this paper.

Subperiod	1	2	3	4
Marginal Interest Costs (Repo Rate)	2 %	2 %	3 %	3%
Remuneration at the Average Repo Rate:				
Marginal Interest Revenues	2.5 %	2.5 %	2.5 %	2.5 %
Net Marginal Interest Costs	-0.5 %	-0.5 %	0.5 %	0.5 %
Remuneration at the Current Repo Rate:				
Marginal Interest Revenues	2 %	2 %	3 %	3 %
Net Marginal Interest Costs	0 %	0 %	0 %	0 %
No Remuneration:				
Marginal Interest Revenues	0 %	0 %	0 %	0 %
Net Marginal Interest Costs	2 %	2 %	3 %	3 %

Table 1: Marginal Interest Costs and Revenues with Alternative Methods of Paying Interest on Required Reserves

ated or if they are remunerated at the average rate, *net* marginal interest costs of holding reserves will differ over a reserve maintenance period if the repo rate is changed during that period (see table 1). If the minimum reserve system allows for averaging provisions of required reserves, which is the case in the euro area,[3] this will imply that banks shift holdings of required reserves into that part of the reserve maintenance period in which the repo rate is lower. The problem of this reserve shifting is twofold. Firstly, the buffer-function of the minimum reserve system against short-term transitory liquidity shocks, which is the key function of the minimum reserve system in the euro area (European Central Bank, 2004b, p. 79), is impaired. Secondly, it implies that banks are affected differently by a monetary policy impulse (we will comment on this in more detail in section 4). However, if reserves are remunerated at the current repo rate, banks will not have the incentive to shift their holdings of required reserves since net marginal interest costs of holding reserves will be zero over the whole maintenance period as table 1 exemplifies.[4] Consequently, the central bank will be able to change the repo rate within a reserve maintenance period without provoking a problematic reserve shifting. This means

[3]Compliance with the reserve requirements is determined on the basis of the average of the daily balances on the credit institutions' accounts with the Eurosystem over the maintenance period.

[4]Note, that the crucial point is not that net marginal interest costs are zero over the whole maintenance period but that they are constant over the whole maintenance period even if the repo rate is changed.

that the commitment of the Eurosystem's governing council to decide on interest rate changes only at the beginning of a reserve maintenance period, i.e. only during the first of its bi-monthly meetings, will no longer be necessary.[5] Consequently, monetary policy can be conducted more flexibly. One may argue that in practice a more routine flexibility of monetary policy in the euro area is not needed, that, for example, the FOMC of the U.S. Federal Reserve System only meets about every six weeks which seems like enough flexibility. However, reducing the flexibility of monetary policy is costly, and it is advantageous if these costs, even if they should be small, can be avoided.

For showing that remunerating required reserves at the current repo rate enhances the flexibility of monetary policy, we develop two theoretical models which combine the reserve management of credit institutions and the monetary policy activities of a central bank. Both models capture the main institutional features of the euro area but they differ in the way in which reserves are remunerated. The related literature on banks' reserve management has so far focused on the reserve management of U.S. banks and the U.S. federal funds market.[6] However, for the banks' reserve management the institutional framework plays a crucial role (see Bartolini, Bertola, and Prati, 2003; Bartolini and Prati, 2006). Therefore, the results of the papers focussing on the U.S. cannot be easily transferred to the euro area, but specific institutional features of the euro area have to be considered. This has been

[5]The reason for the governing council's commitment, which was announced in November of 2001 (see Duisenberg, 2001), was to avoid speculation on interest rate changes occurring every two weeks which led to under- and overbidding behaviour in several MROs. An MRO will be characterized by overbidding (underbidding) if the aggregated bidding volume exceeds (remains under) the Eurosystem's benchmark allotment. The benchmark allotment is the Eurosystem's assessment of actual liquidity needs of the banking sector, providing smooth provisions of required reserves over a reserve maintenance period (European Central Bank, 2004a, p. 16-18). However, the governing council's commitment did not solve the problem of unbalanced bidding behaviour, therefore, in 2004 the ECB changed the design of its monetary policy instruments. Concerning the changes to the Eurosystem's monetary policy instruments see European Central Bank (2003, 2005). Note that when changing its monetary policy instruments, the Eurosystem assumed that the unbalanced bidding behaviour was triggered by interest rate change expectations (European Central Bank, 2000, 2003). However, two further triggers for the overbidding behaviour have been discussed: an asymmetric objective function of the ECB (Ayuso and Repullo, 2001, 2003) and a flawed-rationing-rule in the ECB's fixed rate tenders (Nautz and Oechssler, 2003, 2006). We will come back to the Nautz-Oechssler-approach in section 4.

[6]See, for example, Ho and Saunders (1985), Hamilton (1996), Clouse and Dow (1999, 2002), Furfine (2000), and Bartolini, Bertola, and Prati (2001, 2002).

done, for example, in several papers analyzing the causes and consequences of the banks' under- and overbidding behaviour in the Eurosystem's MROs.[7] This paper contributes to the literature by analyzing the influence of the remuneration of required reserves on the banks' reserve management and, therefore, by analyzing the importance of this remuneration for the conduct of monetary policy.

The remainder of this paper is structured as follows. Section 2 develops a reserve management model assuming that reserves are remunerated at the current repo rate (current rate model). In section 3, the model is changed by assuming that reserves are remunerated at an average rate (average rate model). Section 4 discusses the implications of our theoretical analysis for the euro area, and section 5 briefly summarizes the paper.

2 Current Rate Model

2.1 Structure

This section presents a reserve management model assuming that holdings of required reserves are remunerated at the current repo rate. Firstly, the liquidity management of a single bank is considered. The bank can cover its liquidity needs either by borrowing from the central bank or in the interbank market where it can also place liquidity. The bank minimizes its net liquidity costs by choosing its optimal central bank borrowing, its optimal transactions in the interbank market, and its optimal intertemporal allocation of required reserves. Secondly, two groups of banks differing in their marginal costs of borrowing liquidity from the central bank are considered. When aggregating over the banks' transactions in the interbank market, the equilibrium interbank market rate is derived so that thirdly, the closed-form solution for the banks' optimal liquidity management can be determined. Finally, the effect of a monetary policy impulse on the banks' minimal liquidity costs is analyzed in order to see in how far banks are affected differently by this policy impulse.

[7] See, for example, Breitung and Nautz (2001), Ayuso and Repullo (2001, 2003) Ewerhart (2002), Bindseil (2002), Nautz and Oechssler (2003, 2006) and Neyer (2004).

2.2 Liquidity Management of a Single Bank

There are two time periods, $t = 1, 2$, which cover a reserve maintenance period. An isolated, price-taking bank is considered which needs liquidity resulting from autonomous factors A and reserve requirements RR imposed by a central bank. Concerning required reserves, the bank can make use of averaging provisions. The reserve requirement is fulfilled if

$$RR = \frac{R_1 + R_2}{2}, \tag{1}$$

with $R_t \geq 0$ being the reserve holdings of the bank in period t.

To cover its liquidity needs, the bank can borrow in each period from the central bank or in the interbank market where it can also place liquidity. Both, central bank credits and interbank market transactions, have a maturity of one period.

Central Bank Credits: The central bank specifies the interest rate l_t (repo rate), and the bank receives the amount of liquidity from the central bank it wishes to borrow from at this rate, i.e. we assume at first that the central bank uses a pure interest rate policy instrument.[8] The loan the bank receives from the central bank is denoted by K_t, with $K_t \geq 0$. This loan has to be based on adequate collateral. We assume that rate of return considerations induce a strict hierarchy of the bank's assets,[9] and that assets which can serve as collateral have a relatively low rate of return due to the specific criteria they have to fulfil.[10] Consequently, there are increasing marginal opportunity costs of holding collateral: The more liquidity the bank borrows from the central bank, the more collateral it must hold at the dispense of assets with a

[8] The Eurosystem uses most of the time an interest rate policy instrument, but on occasion, namely if total bids exceed the Eurosystem's benchmark allotment, it will use a total reserves instrument by rationing liquidity in the form of a pro-rata allotment of the individual bank bids. Therefore, in subsection 3.5, we will also analyze the banks' reserve management for this case.

[9] This approach can be compared with the one in Blum and Hellwig (1995). Blum and Hellwig consider a bank with deposits and equity. The bank can put these funds into loans to firms, government bonds or reserves of high powered money. Blum and Hellwig assume that rate of return considerations induce a strict preference for loans over bonds and for bonds over reserves.

[10] In the euro area, eligible assets have been defined by the Eurosystem. The list of these assets are available on the ECB's website (www.ecb.de). For details concerning the criteria for an asset to be eligible, see European Central Bank (2006, chapter 6)

higher rate of return. This is combined with increasing marginal costs due to the assumed hierarchical order of the bank's assets. Postulating a tractable quadratic form for these opportunity costs, the relevant cost function is given by

$$Q(K_t) = qK_t + \frac{1}{2}K_t^2, \tag{2}$$

with the parameter $q \geq 0$. The parameter q, which reflects the level of marginal opportunity costs of holding collateral, plays a key role in our model.

Interbank Market Transactions: If the bank borrows more funds from the central bank than it needs to cover its own liquidity needs, it will place the excess liquidity at the rate e_t in the interbank market. If, on the other hand, the loan from the central bank is too small to cover the bank's total liquidity needs, it will borrow at the rate e_t in the interbank market. The bank's position in the interbank market is given by

$$B_t = A + R_t - K_t \lesseqgtr 0. \tag{3}$$

Trading in the interbank market, the bank faces transaction costs which are given by

$$Z(B_t) = \frac{1}{2}\left(B_t\right)^2. \tag{4}$$

Equation (4) represents a common approach of modelling transaction costs in the interbank market (see, for example, Campbell, 1987; Bartolini, Bertola, and Prati, 2001). The quadratic form reflects increasing marginal costs of searching for banks with matching liquidity needs and those resulting from the need to split large transactions into many small ones to work around credit lines.

Holdings of required reserves are remunerated at the end of each period t at the current repo rate l_t. Consequently, net liquidity costs in period t consist of interest payments to the central bank, interest payments or interest yields resulting from transactions in the interbank market, collateral's opportunity costs, transaction costs and interest yields from holding required reserves:

$$C_t(K_t, R_t) = K_t l_t + B_t(R_t, K_t)e_t + Q(K_t) + Z(B_t(R_t, K_t)) - R_t l_t. \tag{5}$$

Note that reserves are remunerated at the end of each period at the current repo rate. The bank minimizes its net total liquidity costs over the maintenance period by choosing the optimal intertemporal allocation of required reserves and optimal borrowing from the monetary authority. Disregarding discounting, whose impact is negligible over this short horizon, the bank's objective function becomes

$$\min_{K_t,R_t} \left\{ \sum_{t=1}^{2} C_t(K_t, R_t) \right\}. \tag{6}$$

Since the bank can make use of averaging provisions to meet its reserve requirements, it faces a simple dynamic optimization problem. Defining V_t as the associated value function and replacing R_2 by $(2RR - R_1)$, the Bellman equation for the intra-maintenance period optimization problem is given by

$$V_1 = \min_{K_1,R_1} \{C_1(K_1, R_1) + V_2\} \quad \text{subject to} \quad K_t, R_t \geq 0. \tag{7}$$

Solving this optimization problem leads us to the following first order conditions:

$$-e_t + l_t + K_t + q - B_t - \lambda_t = 0, \tag{8}$$

$$\lambda_1 K_t = 0, \quad \lambda_t \geq 0, \quad K_t \geq 0, \tag{9}$$

$$e_1 + B_1(K_1, R_1) - l_1 = e_2 + B_2(K_2^{opt}(R_1), R_1) - l_2. \tag{10}$$

The first order condition for optimal central bank borrowing given in equation (8) states that if the bank covers its liquidity needs in the interbank market *and* at the central bank, marginal costs of interbank market funds $(B_t + e_t)$ will be equated to marginal costs of central bank funds $(q + K_t + l_t)$. If the bank places liquidity in the interbank market, marginal costs of this transaction $(q + K_t + l_t - B_t)$ will equal its marginal benefits e_t. (Note, that in this case $B_t < 0$.) The first order conditions represented by (9) reflect the non-negativity constraint for K_t. The equations (8) and (9) reveal the importance of the bank's opportunity costs of holding collateral.

They determine if and when how much liquidity the bank borrows from the monetary authority and how much liquidity it borrows or places in the interbank market. There are two thresholds, $\bar{q}$ and $\underline{q}$, for the cost parameter q. If $q \geq \bar{q}$, the bank's opportunity costs of holding collateral will be so high that borrowing directly from the central bank will be so expensive that it will prefer to cover its total liquidity needs in that market, i.e. $K_t = 0$. If $q < \underline{q}$, the bank's opportunity costs of holding collateral will be so small that it will borrow more reserves from the central bank than it needs to cover its own liquidity needs in order to place the excess liquidity in the interbank market, i.e. $K_t > (A + R_t)$. Equation (10) gives the first order condition for the optimal intertemporal allocation of reserves. It requires *net* marginal costs of holding required reserves to be the same in both periods. Marginal costs of holding reserves are presented by the first two terms on each side of the equation, the last term on each side captures marginal revenues of holding reserves which result from their remuneration. Since in this model the requirement $R_t \geq 0$ is not a binding constraint, we neglect this constraint in the Lagrangian.

The first order conditions allow us to determine the bank's optimal liquidity management which consists of its optimal intertemporal allocation of required reserves $R_t(e_1, e_2)$, its optimal borrowing from the central bank $K_t(e_1, e_2)$, and its optimal transactions in the interbank market $B_t(e_1, e_2)$. These provisional results (they are provisional because they still depend on the endogenous variable e_t) are presented in appendix B. They are used to determine the equilibrium interbank market rate e_t^* in the next subsection which then allows us to present the closed-form solution for the bank's optimal liquidity management.

2.3 Interbank Market

In the euro area, banks face different marginal opportunity costs of holding collateral. Firstly, marginal costs of holding collateral vary across countries within the euro area due to differences in the financial structure across Member States of the EMU (Hämäläinen, 2000). Secondly, banks tend to focus on different business segments. As a consequence of this specialization their asset structures will be distinct from one another, implying that they have different marginal opportunity costs of holding collateral. For capturing this aspect, we assume that there are two groups of price-taking banks differing in their marginal opportunity costs of holding collateral q. For each group, we consider a representative bank, bank A and bank B, with $q^A < \underline{q}_t \; \forall t$ and $q^B \geq \bar{q}_t \; \forall t$, i.e. bank A always places liquidity in the interbank market, while bank B always satisfies its total liquidity needs in that market.[11] Then solving

$$B_t^{A,opt} + B_t^{B,opt} = 0 \tag{11}$$

for e_t ($B_t^{A,opt}$ is given by the first line of the equations (43) and (44) in appendix B, $B_t^{B,opt}$ by the second line of these equations), we learn that the equilibrium interbank market rate is given by

$$e_t^* = l_t + q^A + 3(A + RR). \tag{12}$$

The equilibrium interbank market rate e_t^* reflects bank A's marginal costs of placing liquidity in the interbank market and is therefore determined by bank A's marginal interest costs l_t, its marginal opportunity costs of holding collateral ($q^A + K_t^{A,opt} = q^A + 2(A + RR)$) and its marginal costs of placing liquidity in the interbank market ($-B_t^{A,opt} = A + RR$).[12]

Equation (12) reveals that there is a positive spread between the interbank market rate e_t^* and the repo rate l_t. As in Neyer and Wiemers (2004) the positive spread is the result of the heterogeneous banking sector. This heterogeneity results from the banks' different opportunity costs of holding collateral. They imply that a kind of intermediation occurs. The banks with relatively low opportunity costs of holding collateral (represented by bank A) borrow more reserves from the central bank than they need to cover their own liquidity needs to lend the excess liquidity via the interbank market to those banks with relatively high opportunity costs (represented by bank B). The spread reflects bank A's transaction costs of placing liquidity in the interbank market and its opportunity costs of holding the collateral necessary for borrowing the respective liquidity from the central bank. It should be noted that

[11] We have also solved the model by assuming a continuum of banks differing in q. The results are the same as in the two-bank case presented in this paper but their presentation is more complex.

[12] The equilibrium values for $K_t^{A,opt}$ and $B_t^{A,opt}$ are given by the equations (14) and (16).

we focus on different opportunity costs of holding collateral. However, the crucial point for this kind of intermediation to occur, and therefore for the positive spread, is that banks have different costs of obtaining funds directly from the central bank, which, for example, may also result from different operating costs.[13]

The outcome of the current rate model concerning the interbank market rate is summarized by

> *Result 1: The interbank market rate in period t reflects the banks' marginal costs of placing liquidity in the interbank market. Consequently, it is determined by marginal transaction costs in the interbank market, the repo rate in period t, and marginal opportunity costs of holding collateral. The positive spread between the interbank market rate and the repo rate is the result of a heterogeneous banking sector.*

2.4 The Banks' Optimal Liquidity Management

Inserting e_t^* given by equation (12) into the provisional results given in appendix B, the final results for the banks' optimal liquidity management are obtained. The optimal intertemporal allocation of required reserve holdings is given by

$$R_1^{A,opt} = R_2^{A,opt} = R_1^{B,opt} = R_2^{B,opt} = RR, \tag{13}$$

which gives us

> *Result 2: In the current rate model, all banks provide their reserve requirements smoothly over a reserve maintenance period, no bank post-*

[13]In the euro area, there is a positive spread between the interbank market rate and the repo rate. For respective empirical analyses see, for example, Nyborg, Bindseil, and Strebulaev (2002), Ayuso and Repullo (2003), Ejerskov, Moss, and Stracca (2003), Nyborg, Bindseil, and Strebulaev (2002), and Neyer and Wiemers (2004). The explanation in this paper for the observed positive spread is a heterogeneous banking sector, banks have different costs when borrowing liquidity directly from the central banks so that a kind of intermediation occurs (for details see Neyer and Wiemers, 2004). An alternative explanation is given by Ayuso and Repullo (2003). In their article, the positive spread supports their hypothesis of an asymmetric objective function of the Eurosystem in the sense that the Eurosystem, which wants to steer the interbank rate towards a target rate, is more concerned about letting the interbank rate fall below the target.

pones or frontloads reserves, irrespectively of a change in the repo rate within the reserve maintenance period. Obviously, this implies also on aggregate smooth provisions of required reserves.

A driving force behind this result is that required reserves are remunerated at the current repo rate. This implies that holding reserves is neutral with regard to interest payments to and interest revenues from the central bank. However, nevertheless banks are not indifferent regarding their intertemporal allocation of required reserves. Due to the convex form of the opportunity cost function it is optimal for bank A to hold in both periods the same amount of reserves, and due to the convex form of the transaction cost function it is also optimal for bank B to fulfill its reserve requirements smoothly.

The smooth provisions of required reserves imply that the banks' liquidity needs are the same in both periods. Consequently, the banks' optimal central bank borrowing and their optimal transactions in the interbank market are the same in both periods too:

$$K_1^{A,opt} = K_2^{A,opt} = 2(A + RR), \tag{14}$$

$$K_1^{B,opt} = K_2^{B,opt} = 0, \tag{15}$$

$$B_1^{A,opt} = B_2^{A,opt} = -(A + RR), \tag{16}$$

and

$$B_1^{B,opt} = B_2^{B,opt} = A + RR. \tag{17}$$

Analogously to the ECB, we define the banks' aggregate liquidity demand at the central bank in the first period which allows on aggregate for smooth provisions of required reserves over the maintenance period as the central bank's benchmark amount (see page 4). In our model, aggregate liquidity demand at the central bank consists only of bank A's demand for reserves which is equal to $2(A+RR)$ as shown by equation (14). Since this is the amount of liquidity which allows for smooth provisions of aggregate required reserves over the maintenance period, we obtain

Result 3: In the current rate model, aggregate liquidity demand at the central bank does not deviate from the central bank's benchmark amount.

2.5 Are Banks Affected Differently by a Monetary Policy Impulse?

Next, we will analyze whether banks are affected differently by a monetary policy impulse in the form of a change in the repo rate. The idea is to look at the change in the banks' minimal liquidity costs. If the repo rate is raised or cut at the beginning of the maintenance period, the change in the banks' net minimal liquidity costs will be given by

$$\frac{\partial (V_1|l_1 = l_2 = l)^A}{\partial l} = \frac{\partial (V_1|l_1 = l_2 = l)^B}{\partial l} = 2A. \tag{18}$$

For analyzing the consequences of a change in the repo rate within the reserve maintenance period on the banks' minimal liquidity costs, we look at the difference between the minimal costs without and with a change in the repo rate. This difference is given by

$$(V_1|l_2 = l_1)^A - (V_1|l_1 \neq l_2)^A = (V_1|l_2 = l_1)^B - (V_1|l_1 \neq l_2)^B = A(l_1 - l_2). \tag{19}$$

Both equations show that the impact of this monetary policy impulse on the banks' minimal liquidity costs is the same for both banks which leads us to

Result 4: In the current rate model, banks are not affected differently by a monetary policy impulse in the form of a change in the repo rate.

The reason for this is that the change in the repo rate implies only a change in the liquidity costs resulting from *autonomous factors*. This change is the same for both banks since the repo rate and the interbank market rate change to the same extent as equation (12) shows.[14] Due to the remuneration of reserves at the current repo rate, the net interest costs of holding *required reserves* are not influenced by a monetary policy impulse in the form of a change in the repo rate. Holding required reserves is neutral with regard to interest payments to and interest revenues from the central bank.

3 Average Rate Model

3.1 Liquidity Management of a Single Bank

In the following, we change the current rate model by assuming that reserves are remunerated at the average of the repo rates l_1 and l_2 at the end of the second period instead of at the current repo rate l_t at the end of each period. Then, liquidity costs in period t are given by

$$C_t(K_t, R_t) = K_t l_t + B_t e_t + Q(K_t) + Z(B_t) - 2RR\left(\frac{l_t + l_{t-1}}{2}\right) I_{[t=2]}. \qquad (20)$$

The only difference to equation (5) is the last term which shows that required reserves are remunerated at the average of l_1 and l_2. The indicator function $I_{[t=2]}$ takes a value of 1 when $t = 2$, and 0 otherwise. This reflects that interest is paid at the end of the maintenance period. Analogously to the current rate model, the bank minimizes total liquidity costs over the maintenance period, while keeping average reserves over the maintenance period to the required level RR by choosing the optimal intertemporal allocation of required reserves and optimal borrowing from the monetary authority. The only difference in the first order conditions concerns the first order condition for the intertemporal allocation of required reserves which is now given by

$$e_1 + B_1(K_1, R_1) = e_2 + B_2(K_2^{opt}(R_1), R_1). \qquad (21)$$

[14]The repo rate is the relevant interest rate for bank A which borrows the liquidity directly from the central bank, and the interbank market rate is the relevant interest rate for bank B which covers its total liquidity needs in that market.

As in the current rate model, the first order condition requires that net marginal costs of holding reserves must be the same in both periods. However, the difference is that in this model, interest yields of holding reserves play no role (compare equations (10) and (21)). The reason for this is that due to the remuneration of reserves at the average of l_1 and l_2, marginal revenues of holdings reserves are the same in both periods so that they are irrelevant for intertemporal optimization.

3.2 Interbank Market

As in the current rate model, the first order conditions allow us to determine the provisional results for the bank's optimal liquidity management (see appendix C) which we use to determine the equilibrium interbank market rate. Analogously to the current rate model, we learn by solving equation (11) for e_t, where $B_t^{A,opt}$ and $B_t^{B,opt}$ are given in appendix C, that the equilibrium interbank market rate is given by

$$e_1^* = e_2^* = 3(A + RR) + q^A + \frac{l_1 + l_2}{2}. \tag{22}$$

As in the current rate model, e_t^* reflects bank A's marginal costs of placing liquidity in the interbank market. The influence of A, RR, and q^A is the same as in the current rate model. However, the influence of the repo rate differs. In the current rate model, the interbank market rate in period t only depends on the repo rate of the same period as shown by equation (12). In the average rate model, on the other hand, the interbank market rate in period t depends on the repo rate of both periods as revealed by equation (22). This implies that the interbank market rate will be smoothed if the repo rate is changed within the reserve maintenance period: If the central bank raises (cuts) the repo rate in the second period, the interbank market rate will already increase (decrease) in the first period, and the increase (decrease) in the second period will be dampened.[15] Responsible for this smoothing effect is the remuneration of required reserves at the average rate. It implies that bank A's marginal costs and its marginal benefits of holding reserves

[15]In the current rate model, $\partial e_1^*/\partial l_2 = 0$ and $\partial e_2^*/\partial l_2 = 1$ as equation (12) reveals. In the average rate model, $\partial e_1^*/\partial l_2 = 1/2 > 0$ and $\partial e_2^*/\partial l_2 = 1/2 < 1$ as equation (22) shows.

will diverge if the central bank changes the repo rate within a maintenance period (see table 1) which implies that bank A will shift reserves into that part of the reserve maintenance period in which the repo rate is lower as shown formally in the next subsection. Let us assume that the central bank raises the repo rate so that $l_1 < l_2$. Then, bank A frontloads required reserves, i.e. its liquidity needs increase in the first period. Consequently, it borrows more liquidity from the central bank in that period. This means that, due the convex form of the opportunity cost function, its marginal opportunity costs of holding collateral, and, therefore, its marginal costs of placing liquidity in the interbank market and, therefore, the interbank market rate increase. In the second period, on the other hand, bank A's liquidity needs decrease. Consequently, it borrows less liquidity from the central bank which implies that its marginal opportunity costs of holding collateral decrease. This again has a dampening effect on its marginal costs of placing liquidity in the interbank market, and, therefore on the interbank market rate.

Equation (22) shows that the interbank market rate is not only smoothed but that it is always the same in both periods. The reason for this is that intertemporal optimality requires *net* marginal costs of holding reserves to be the same in both periods. Since marginal benefits of holding reserves are the same (due to the remuneration of reserves at the average rate, they are in both periods $(l_1 + l_2)/2$), also marginal costs of holding reserves must be the same. Consequently, bank A frontloads or postpones so much reserves until the resulting different marginal opportunity costs of holding collateral balance the different marginal interest costs in the two periods. The interbank market rate reflects bank A's marginal costs of placing liquidity in that market. These costs consist of bank A's marginal costs of borrowing liquidity from the central bank and of its marginal transaction costs. Bank A's marginal transaction costs are the same in both periods since the amount of liquidity bank A places in the interbank market does not change, it is always equal to $(A + RR)$ as we will see in the next subsection. Bank A's marginal costs of borrowing liquidity from the central bank correspond to its marginal costs of holding reserves, and since the latter are the same in both periods as argued above, the equilibrium interbank market rate must be identical in both periods.[16]

[16]Note that in the current rate model, things are different: The crucial point is that marginal benefits

The outcome of the average rate model concerning the interbank market rate is summarized by

> *Result 5: In the average rate model, there is a smoothing effect on the interbank market rate: the interbank market rate already decreases (increases) before the central bank actually cuts (raises) the repo rate.*

3.3 The Banks' Optimal Liquidity Management

Analogously to the current rate model, we have inserted e_t^* given by equation (22) into the provisional results given in appendix C in order to determine the final results for the banks' optimal liquidity management.

The optimal intertemporal allocation of required reserves is given by

$$R_1^{A,opt} = RR - \frac{l_1 - l_2}{2}, \tag{23}$$

$$R_2^{A,opt} = RR + \frac{l_1 - l_2}{2}, \tag{24}$$

and

$$R_1^{B,opt} = R_2^{B,opt} = RR. \tag{25}$$

These equations show that contrary to the current rate model, a change in the repo rate influences bank $A's$ optimal intertemporal allocation of required reserves. If the repo rate is cut, bank A will postpone required reserves ($R_1^{A,opt} < R_2^{A,opt}$), and if the repo rate is raised, it will frontload reserves ($R_1^{A,opt} > R_2^{A,opt}$). The driving

of holding reserves (l_t) will differ in the two periods if the interest rate is changed. Consequently, intertemporal optimality requires marginal costs of borrowing from the central bank to be different too, so that net marginal costs of holding reserves are the same in both periods. Obviously, this will be the case if the repo rate is changed (without reserve shifting). Since marginal costs of central bank borrowing will differ in both periods if the repo rate is changed, also marginal costs of placing liquidity in the interbank market and, therefore, also the interbank market rates will be different in the two periods.

force behind this result is that reserves are remunerated at the average of l_1 and l_2. This remuneration implies that bank A's *net* marginal interest costs will differ over a reserve maintenance period if the repo rate is changed (see table 1) so that bank A will shift holdings of required reserves into that part of the maintenance period in which the repo rate is lower.

This is not the case for bank B. Equation (25) shows that, independently of a change in the repo rate, bank B provides its required reserves smoothly over the maintenance period. The reason for this is that for bank B, marginal interest costs and marginal interest revenues of holding reserves are the same in both periods, even if the central bank cuts or raises the repo rate. Reserves are remunerated at the average of l_1 and l_2, so that marginal revenues of holding reserves in both periods are $(l_1 + l_2)/2$; and bank B's marginal costs of holding reserves are equal to e_t^* which is also the same in both periods (see equation (22) and comments on page 16). Consequently, bank B has no incentive to postpone or frontload required reserves. On the contrary, it is optimal to provide the reserves smoothly: Bank B covers its liquidity needs exclusively in the interbank market, and transactions in the interbank market involve increasing marginal transaction costs which implies that it is optimal to transact the same amount in both periods.

Bank B's smooth provisions of required reserves and bank A's postponing or front-loading of required reserves leads us to

> *Result 6: In the average rate model, aggregate required reserves will be provided unevenly over the reserve maintenance period if the repo rate is changed within that maintenance period.*

The uneven provisions of required reserves of bank A are also reflected by its central bank borrowing which is given by

$$K_1^{A,opt} = 2(A + RR) - \frac{l_1 - l_2}{2}, \tag{26}$$

and

$$K_2^{A,opt} = 2(A + RR) + \frac{l_1 - l_2}{2}. \tag{27}$$

If bank A postpones required reserves, i.e. if the repo rate is cut, its liquidity needs in the first period are smaller than those in the second period so that $K_1^{A,opt} < K_2^{A,opt}$. Analogously, $K_1^{A,opt} > K_2^{A,opt}$, if the central bank raises the repo rate. Consequently, aggregate central bank borrowing in the first period will lie below the central bank's benchmark which is equal to $2(A + RR)$, while it will exceed the benchmark if the repo rate is raised. This leads us to

> *Result 7: In the average rate model, aggregate central bank borrowing will deviate from the central bank's benchmark amount if the repo rate is changed within the reserve maintenance period.*

Since bank B provides its required reserves smoothly its liquidity needs are the same in both periods. Consequently, the amount of liquidity transacted in the interbank market is also the same in both periods:

$$B_1^{A,opt} = B_2^{A,opt} = -(A + RR), \tag{28}$$

$$B_1^{B,opt} = B_2^{B,opt} = A + RR. \tag{29}$$

3.4 Are Banks Affected Differently by a Monetary Policy Impulse?

For analyzing the consequences of a change in the repo rate within the reserve maintenance period on the banks' minimal liquidity costs,[17] we look again at the difference between the minimal costs without and with a change in the repo rate. This difference is given by

$$(V_1|l_2 = l_1)^A - (V_1|l_1 \neq l_2)^A = A(l_1 - l_2) + \frac{(l_1 - l_2)^2}{4} \tag{30}$$

[17] The consequences of a change in the repo rate at the beginning of a reserve maintenance period, i.e. in the first period so that $l_1 = l_2$, are the same as in the current rate model (see page 13) since the only difference between the design of this average rate model and of the current rate model is that reserves are remunerated at the average of l_1 and l_2 and this difference will obviously not be effective if l_1 is equal to l_2.

and

$$(V_1|l_2 = l_1)^B - (V_1|l_1 \neq l_2)^B = A(l_1 - l_2). \tag{31}$$

These equations lead us to

Result 8: In the average rate model, banks are affected differently by a monetary policy impulse in form of a change in the repo rate within the reserve maintenance period.

Independently of whether the repo rate is cut or raised the fraction in equation (30) has a positive sign. This means that in case the repo rate is cut, bank A's minimal liquidity costs decrease more than bank B's and in case the monetary authority raises the repo rate, bank A's minimal liquidity costs increase less or even decrease. The reason is obvious: bank A benefits from frontloading or postponing its required reserves.

3.5 Rationing

So far, we have assumed that the central bank always totally satisfies the banks' demand for reserves, even if the aggregate demand exceeds the central bank's benchmark amount. However, in the euro area, the ECB will normally ration liquidity in form of a pro-rata allotment of the individual bank bids if total bids exceeds the ECB's benchmark allotment (European Central Bank, 2004b, p. 80).[18] Therefore, this subsection analyzes the optimal liquidity management of bank A and bank B assuming that the central bank does not provide more liquidity than its benchmark amount, i.e. that it will ration liquidity if the demand exceeds its benchmark. Since this will only be the case if the central bank is going to *raise* the repo rate, we can restrict our analysis to this scenario.

[18] In effect, this case is one where the Eurosystem switches from an interest rate policy instrument to a total reserves policy instrument. Note that this paper does not aim at analyzing and discussing the use of this kind of hybrid forms of monetary policy operations in general but for both kinds of instruments the consequences on the banks' reserve management are analyzed within our model framework.

In this two-bank case, the central bank's benchmark amount in each period is equal to $2(A+RR)$, i.e. $K_1^A = \min[K_1^{A,opt}, 2(A+RR)]$. Solving the average rate model under the rationing assumption, the banks' optimal liquidity management is given by

$$K_1^A = K_2^{A,opt} = 2(A+RR), \tag{32}$$

$$B_1^{A,opt} = B_2^{A,opt} = -(A+RR), \tag{33}$$

$$K_1^{B,opt} = K_2^{B,opt} = 0, \tag{34}$$

$$B_1^{B,opt} = B_2^{B,opt} = A+RR, \tag{35}$$

$$R_1^{A,opt} = R_2^{A,opt} = R_1^{B,opt} = R_2^{B,opt} = RR. \tag{36}$$

These results show that in both periods the benchmark amount of liquidity is provided via bank A to the banking sector, that in both periods the same amount of liquidity $(A+RR)$ is transacted in the interbank market, and, despite the increase in the repo rate, aggregate required reserves are provided smoothly over the reserve maintenance period.

However, the extent to which the banks are affected differently by the monetary policy impulse is higher when compared to the non-rationing case: If there is no rationing, bank A's liquidity costs will increase less than bank B's, because bank A benefits from the frontloading of its reserve holdings (see page 19). If, on the other hand, the central bank rations liquidity, bank B faces even additional liquidity costs while bank A's liquidity costs actually decrease:

$$V_1^A(l_2 = l_1) - V_1^A(l_1 < l_2) = -(l_1 - l_2)RR > 0 \tag{37}$$

$$V_1^B(l_2 = l_1) - V_1^B(l_1 < l_2) = (l_1 - l_2)(2A + RR) < 0. \tag{38}$$

Consequently, the extent to which the banks are affected differently is higher than in the non-rationing case. The reason is that the rationing implies an additional increase in the interbank market rate from which bank A as a lender benefits and bank B as a borrower suffers. The explanation for the additional increase in the market rate is as follows. The interbank market rate reflects bank A's marginal costs of placing liquidity in the interbank market which consist inter alia of interest payments to the central bank,[19] and the rationing implies an additional increase in the marginal interest payments to the central bank since in both periods, they are determined only by the (higher) repo rate l_2:

$$e_1^* = e_2^* = l_2 + q_i + 3(A + RR) = MC^A. \tag{39}$$

The intuition for this result is as follows. If bank A wants to place additional liquidity in the interbank market in the first period, it cannot borrow the necessary liquidity from the central bank due to rationing. Consequently, bank A has to reduce its reserve holdings R_1^A. However, this implies that bank A has to hold more reserves in the second period to fulfil its reserve requirements which again implies that it has to borrow more reserves from the central bank in the second period - at the higher rate l_2. Formally, the additional increase in the interbank market rate due to the rationing can be seen by comparing the differentiation of the interbank market rate with respect to l_2 with and without rationing. Without rationing $\partial e_t^*/\partial l_2 = 0.5 \forall t$ (see equation (22)), with rationing $\partial e_t^*/\partial l_2 = 1 \forall t$.

The consequences of the central bank's rationing of liquidity are summarized by

> *Result 9: If the central bank raises the repo rate in the second period and rations liquidity by providing only its benchmark amount in the first period, aggregate required reserves will be provided smoothly over the reserve maintenance period. However, the rationing implies that the extent*

[19] For more detailed information see analogously the explanations concerning the interbank market rate given given on page 15.

to which the banks are affected differently by a monetary policy impulse is higher than in the non-rationing case.

4 Implications for the Euro Area

The main implication of this analysis for the euro area is that if reserves were remunerated at the current repo rate instead of at the average, over the maintenance period, of the repo rates, monetary policy could be conducted more flexibly since the commitment of the Eurosystem's governing council to change interest rates only at the beginning of a month would no longer be necessary in order to avoid a problematic reserve shifting.

The reserve shifting is problematic for two reasons. Firstly, it reduces the buffer-function of the minimum reserve system against short-term transitory liquidity shocks which has been one of the main reasons for implementing a minimum reserve system in the euro area (European Central Bank, 2004b, p. 79). Secondly, it implies that banks are affected differently by a monetary policy impulse because they benefit to a different extent from the reserve shifting due to their different marginal opportunity costs of holding collateral.[20] One may argue that this should not a be problem since banks could adjust their asset structure, which is the underlying cause for their different marginal opportunity costs of holding collateral (see page 9), and therefore, for their different exposures to monetary policy impulses. However, if banks have different opportunity costs of holding collateral because of the specific financial structure of their home country, banks may not be able to adjust their asset structure. Then, they will be affected differently by a monetary impulse simply because they are located in different EMU Member Countries. This violates the Eurosystem's principle of equal treatment.[21] If banks have different marginal

[20]In order to implement a reserve shifting strategy, banks must borrow liquidity from the central bank. For these loans banks have to deposit collateral. As the marginal opportunity costs of holding collateral vary across banks in the euro area, also their profitability of the reserve shifting varies, leading to a different decrease/increase in their liquidity costs as a result of a monetary policy impulse.

[21]The Eurosystem's principle of equal treatment specifies that monetary policy instruments must be designed in such a way that credit institutions are treated equally irrespective of their size and their location in the euro area (European Central Bank, 2004b, p. 72).

opportunity costs of holding collateral because of their specialization on specific business segments, the banks can in principle adjust their asset structure, but then they will align their business activities to the monetary policy instruments, i.e. the banks' resource allocation will be influenced by the design of these instruments. This contradicts the idea that these instruments should not hinder the efficient allocation of resources in the euro area.

Furthermore, it is worth mentioning that the reserve shifting implies under- and overbidding behaviour in the MROs: If the ECB cuts interest rates within the maintenance period, banks will postpone reserve holdings implying that the MROs in the maintenance period prior to the interest rate change will be characterized by underbidding. Conversely, in case of an interest rate increase, banks will frontload reserves, i.e. the MROs before the increase in the repo rate will be characterized by overbidding. In the overbidding case, the ECB can avoid the reserve shifting by providing only its benchmark allotment, i.e. by rationing liquidity. However, according to our model such a behaviour increases the extent to which banks are affected differently by a monetary policy impulse. Furthermore, the rationing may trigger the explosion of the bidding process described by Nautz and Oechssler (2003).[22]

Consequently, remunerating required reserves in the euro area at the current repo rate instead of at the average repo rate would have the advantage that monetary policy could be conducted more flexibly since the repo rate could also be changed within a reserve maintenance period without provoking a problematic reserve shifting. A possible advantage of remunerating reserves at the average rate lies in the

[22] It should be noted that although aggregate demand for liquidity will exceed the central bank's benchmark amount if the repo rate is raised, the average-rate model cannot explain - and it has not been the aim of our paper to do so - the extreme overbidding behaviour in the Eurosystem's MROs observed in 2000. For a possible explanation for the extreme and over time increasing overbidding behaviour, we refer the reader to Nautz and Oechssler (2003). In the Nautz-Oechssler-model, each bank can cover its liquidity needs by participating in a central bank's auction and borrowing liquidity at a fixed rate. If a bank bids its true demand and receives this amount, the bank will realize its cost minimum. There are two crucial assumptions: Firstly, banks are boundedly rational players. They have adaptive expectations. Secondly, banks are rationed, i.e. the allotment quota is strictly smaller than one because the central bank is not willing to satisfy the banks' true liquidity demand: By assumption $A_t < D_t$, where A_t is the total allotment and D_t total *true* demand (not total bids). Under these assumptions, Nautz and Oechssler show that under these assumptions the bidding process explodes. (Note, that a possible solution to this problem may be not to ration but always to allot the amount of liquidity the banking sector bids for and to absorb possible excess liquidity via fine-tuning operations.)

smoothing effect on the interbank market rate. However, this effect will only occur if the repo rate is changed within the reserve maintenance period, i.e. at present this advantage is not even used because of the governing councils' commitment, and making use of this advantage, a problematic reserve shifting has to be accepted. It is noteworthy that this paper provides a general argument for paying interest on reserves. If required reserves are not remunerated, same results as in the average rate model will be obtained. Therefore, remunerating reserves (at the current repo rate) increases the flexibility of monetary policy.

5 Summary

We have developed a theoretical model which captures main institutional features of the euro area to analyze the influence of the method of remunerating reserves on the reserve management of credit institutions. We have distinguished between remunerating required reserves at the average, over a maintenance period, of the repo rates and at the current repo rate. It has been shown that if required reserves are remunerated at the average repo rate, the central bank will induce a problematic reserve shifting if it changes the interest rate within the maintenance period because net marginal interest costs will differ over the maintenance period. However, if required reserves are remunerated at the current repo rate, net marginal interest costs of holding reserves will be constant, and the banks do not have an incentive to shift their reserve holdings. In the euro area, required reserves are remunerated at the average repo rate. Considering the results of the theoretical analysis in this paper, monetary policy in the euro area could be conducted more flexibly if required reserves were remunerated at the current repo rate instead since then, the Eurosystem's governing council's commitment to decide on interest rate changes only at the first of its bi-monthly meetings in order to avoid a problematic reserve shifting would no longer be necessary to avoid a problematic reserve shifting. If reserves are not remunerated, the same problems as at their remuneration at the average rate will occur. Therefore, this paper provides a general argument for paying interest on reserves (at the current repo rate): a higher flexibility of monetary policy.

Appendix A: Calculation of the Remuneration of Holdings of Required Reserves

In the euro area, holdings of required reserves are remunerated at the average, over the maintenance period, of the ECB's rate on the main refinancing operations. For the calculation of the remuneration the following formula is used:

$$R_t = \frac{H_t \cdot n_t \cdot r_t}{100 \cdot 360}$$

$$r_t = \sum_{i=1}^{n_t} \frac{MR_i}{n_t},$$

where

R_t = remuneration to be paid on holdings of required reserves for the maintenance period t.

H_t = average daily holdings of required reserves for the maintenance period t.

n_t = number of calendar days in the maintenance period t.

r_t = rate of remuneration on holdings of required reserves for the maintenance period t. Standard rounding of the rate of remuneration to two decimals shall be applied.

i = ith calendar day of the maintenance period t.

MR_i = marginal interest rate for the most recent main refinancing operation settled on or before calendar day i.

Source: European Central Bank (2006, p. 62).

Appendix B: Provisional Results for a Bank's Optimal Liquidity Management in the Current Rate Model

The first order conditions given by the equations (8) to (10) lead to the following provisional results for the bank's optimal liquidity management:

$$R_1^{opt} = RR - \frac{e_1 - e_2 - l_1 + l_2}{2} \quad \forall q, \tag{40}$$

$$R_2^{opt} = RR + \frac{e_1 - e_2 - l_1 + l_2}{2} \quad \forall q, \tag{41}$$

$$K_1^{opt} = K_2^{opt} = \begin{cases} \frac{A+RR-q}{2} + \frac{e_1+e_2-l_1-l_2}{4} & \text{if} \quad q < \bar{q} \\ 0 & \text{if} \quad q \geq \bar{q}, \end{cases} \tag{42}$$

$$B_1^{opt} = \begin{cases} \frac{A+RR+q}{2} - \frac{3(e_1-l_1)-(e_2-l_2)}{4} & \text{if} \quad q < \bar{q} \\ A + RR - \frac{(e_1-l_1)-(e_2-l_2)}{2} & \text{if} \quad q \geq \bar{q}, \end{cases} \tag{43}$$

and

$$B_2^{opt} = \begin{cases} \frac{A+RR+q}{2} + \frac{(e_1-l_1)-3(e_2-l_2)}{4} & \text{if} \quad q < \bar{q} \\ A + RR + \frac{(e_1-l_1)-(e_2-l_2)}{2} & \text{if} \quad q \geq \bar{q}, \end{cases} \tag{44}$$

where

$$\bar{q} = A + RR + \frac{e_1 + e_2 - l_1 - l_2}{2}. \tag{45}$$

Appendix C: Provisional Results for a Bank's Optimal Liquidity Management in the Average Rate Model

In case there is no interest rate change, one obviously obtains the same results as in the current rate model since the only difference to the current rate model is that reserves are remunerated at the average of the repo rates l_1 and l_2 instead of at the current repo rate. Consequently, if there is no change in the repo rate, the difference in the method of remunerating required reserves will be irrelevant for the bank's optimal liquidity management. However, if the repo rate is changed, the bank's optimal behaviour will be different from that in the current rate model. One aspect is that the upper threshold $\bar{q}$ falls apart in the two periods ($\bar{q}_1 \neq \bar{q}_2$): In case the repo rate is cut, the bank may borrow from the central bank in the second but not in the first period: $\bar{q}_{1,c} \leq q < \bar{q}_{2,c}$ (the subscript c stands for *cut*). In case the repo rate is raised, the bank may borrow from the central bank in the first, but not in the second period: $\bar{q}_{2,r} \leq q < \bar{q}_{1,r}$ (the subscript r stands for *raise*). For the sake of simplicity we have excluded these cases in our analysis and have considered only the cases in which $q < \bar{q}_1, \bar{q}_2$ and $q \geq \bar{q}_1, \bar{q}_2$, i.e. only those cases in which a bank borrows either in both periods (bank A) or in no period (bank B) liquidity directly from the central bank (see page 10). Then, the first order conditions given by (8), (9), and (21) lead to the following provisional results for the bank's optimal liquidity management:

$$R_1^{A,opt} = RR - \frac{e_1 - e_2 + l_1 - l_2}{2}, \tag{46}$$

$$R_1^{B,opt} = RR - \frac{e_1 - e_2}{2}, \tag{47}$$

$$R_2^{A,opt} = RR + \frac{e_1 - e_2 + l_1 - l_2}{2}, \tag{48}$$

$$R_2^{B,opt} = RR + \frac{e_1 - e_2}{2}, \tag{49}$$

$$K_1^{A,opt} = \frac{A + RR - q}{2} + \frac{e_1 + e_2 - 3l_1 + l_2}{4}, \tag{50}$$

$$K_1^{B,opt} = 0 \tag{51}$$

$$K_2^{A,opt} = \frac{A + RR - q}{2} + \frac{e_1 + e_2 + l_1 - 3l_2}{4}, \tag{52}$$

$$K_2^{B,opt} = 0 \tag{53}$$

$$B_1^{A,opt} = \frac{A + RR + q}{2} - \frac{3e_1 - e_2 - l_1 - l_2}{4}, \tag{54}$$

$$B_1^{B,opt} = A + RR - \frac{e_1 - e_2}{2}, \tag{55}$$

$$B_2^{A,opt} = \frac{A + RR + q}{2} + \frac{e_1 - 3e_2 + l_1 + l_2}{4}, \tag{56}$$

and

$$B_2^{B,opt} = A + RR + \frac{e_1 - e_2}{2}. \tag{57}$$

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