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Research On Money in the Economy

No. 07-03 – August 2007

## Two-Pillar Monetary Policy and Bootstrap Expectations

Heinz-Peter Spahn

### ROME Discussion Paper Series

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ISSN 1865-7052

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Helpful assistance from Gerhard Wagenhals is gratefully acknowledged. The usual disclaimer applies.

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## **Abstract**

The paper integrates the two-pillar Phillips curve, which explains expected inflation by the money growth trend, within a simple macro model. A Taylor-like interest rule contains also a money growth target. The model takes into account serially correlated supply and money demand shocks; the latter induce goods demand shocks, thereby establishing a feedback mechanism from money to markets which is missing in the modern New Keynesian approach. Two groups of market agents are distinguished from which one derives inflation expectations from money growth trend figures whereas the other builds rational expectations by way of learning. The inspection of output and inflation variances show that a policy of reacting to excess money growth requires precise information on shock characteristics whereas inflation-gap and output-gap oriented interest policies provide more robust stabilization services.

**JEL-Classifikation: E4, E5**

**Keywords:** Money demand shocks, Taylor rule, learning, inflation and output variability

The inclusion of the monetary pillar in the ECB's stated strategy should probably best be seen as an attempt to sing a lullaby as the German public are gently moved from one bed to another.

*Patrick Honohan*<sup>1</sup>

## **I Introduction**

The erection of the famous "two pillars" in the ECB's monetary policy strategy results from admitted severe difficulties of integrating monetary and non-monetary elements into a generalized theory of inflation. This is particularly true, as the traditional IS-LM apparatus, where a policy-controlled money stock variable affects employment and prices, has fallen from favour in academia (Allsopp / Vines 2000). Over the years, the ECB offered various lines of argumentation in favour of this two-pillar strategy, starting from the criterion that monetary policy should be "robust" in the view of model uncertainty (ECB 2000, Issing 2002), and taking up the distinction between short-run and long-run determinants of inflation in later years (ECB 2004: 55-66).

Thus, each pillar of the ECB strategy presents a different analytical approach, and a different time dimension, of the macroeconomic process. Many academic critics remained unconvinced, however, and demanded a more integrated approach or assessed the monetary pillar to be superfluous altogether: if the central bank – by referring to the "real analysis" of the economy, i.e. by exploring tensions between supply and demand on the goods and labour markets – succeeds to control inflation in each short-run period, a separate long-run "monetary analysis" appears to be redundant (e.g. Galí et al. 2004, Woodford 2006, 2007).

In recent years, in a series of papers (Gerlach 2003, 2004, Assenmacher-Wesche / Gerlach 2006), a new approach was presented that claims to offer a synthesis of "real" and "monetary" forces in the macro process. It is a "two-pillar Phillips curve" where expected future inflation is explained by the past money growth trend. Of course, in a monetarist or new classical world, that idea would hardly represent an innovation. Combining the Phillips curve with a

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<sup>1</sup> Quoted from Gerlach (2004: 430).

demand function derived from the quantity theory yields a solution where the (expected) rate of inflation is determined by the money growth rate. But given the modern New Keynesian model setup, where the money supply is identified as endogenous (and therefore usually disappears from the model equations), it turned out to be hard to explain how the quantity of money affects essential macro variables.

Gerlach's main intention is to demonstrate empirically that inflation in the euro area can be decomposed into high- and low-frequency movements that are correlated with monetary growth and the output gap, respectively, a finding that justifies the inclusion of the money growth trend as a shift variable in a Phillips curve equation. It may appear that this also justifies the upholding of the "reference" money growth rate in the ECB's policy concept, where it provides a measure of long-term risks to price stability. But this would be a rash conclusion. Gerlach (2004) finds that money is a useful indicator among others; a separate pillar focused on money is not necessary.

Thus, the existence of "monetarist" inflation expectations among market agents and the effects of pursuing also monetary targets by means of interest rate policies ought to be analyzed more thoroughly, before an assessment of the ECB strategy can be made. The following paper extends Gerlach's work in two ways:

- Whereas he uses a single-equation approach, we envisage a two-pillar Phillips curve as forming a part of a standard macro model where also the money variable is endogenized and different assumptions are explored with regard to the state of information and rationality on the part of market agents.
- The focus is not on empirical questions, but rather on analytical topics of stability of the macro model and on welfare-theoretic aspects of the existence of monetarist beliefs and policies in the model economy. The question is whether the inclusion of a money growth target in a Taylor-like interest rule outperforms a policy strategy that concentrates on the inflation and/or output gap, if the economy is hit by supply, goods demand and money demand shocks.

The program of the remainder of the paper is as follows: Section II summarizes the analytical core of Gerlach's approach and discusses some lines of critique. In Section III, the

two-pillar Phillips curve is supplemented by a simple demand function and an interest rule that describes the behaviour of the central bank. The aim is to explore how the model reacts to supply and money demand shocks. Section IV extends the analysis by allowing two groups of market agents from which one adheres to monetarist beliefs whereas the other builds rational expectations by using all available information. In both these Sections it is found that the model is globally stable, but monetarist beliefs and policies tend to distort efficient adjustment paths after shocks have occurred. Section V concludes.

## II Including Monetarist Expectations in a Phillips Curve

Gerlach (2004) sets up the two-pillar Phillips curve as

$$p_t = \delta p_{t+1}^e + \kappa p_{t-1} + \alpha y_{t-1} + \varepsilon_t^s \quad [1]$$

where  $p_t$  is the inflation rate,  $y_t$  the (log of the) output gap and  $\varepsilon_t^s$  is a supply shock. The approach builds on the assumption of staggered price setting à la Calvo which lets expected future inflation enter the equation<sup>2</sup>; however widely shared beliefs on stylized empirical facts (e.g. Mankiw 2001) point to the inclusion of lagged inflation also (with  $\delta + \kappa = 1$ ). The forward-looking expectation term is determined by the once-lagged money growth trend:<sup>3</sup>

$$p_{t+1}^e = m_{t-1}^T \quad [2]$$

The latter is modelled as a moving average of actual money growth where  $\lambda$  serves as a filter coefficient:

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<sup>2</sup> A modern textbook treatment of the microfoundations of the New Keynesian Phillips curve is Carlin / Soskice (2006: 606-608). See also Mankiw / Reis (2002).

<sup>3</sup> Gerlach also explores two alternative hypotheses, the explanation of future expected inflation by real-growth-rate adjusted money growth and by the inflation trend itself. Econometric support for these hypotheses is somewhat weaker though, particularly for the second alternative. This implies that information on recent money growth rates is not totally embedded in inflation itself, i.e. money figures deliver additional information for prediction. A significant statistical relationship between lagged broad money and output gap has again been found recently, even when lagged values of real interest rates and of the output gap are accounted for (Hafer et al. 2007).

$$m_t^T = \lambda m_t + (1 - \lambda) m_{t-1}^T \quad [3]$$

From the preceding equations, a reduced-form two-pillar Phillips curve can be derived, which, besides the money growth variable, contains once- and twice-lagged output gaps and inflation rates (the derivation for a somewhat simplified model setup will be given in next Section). Econometric tests then show that "while money may not be useful for explaining movements of inflation *around the steady state*, it is helpful for understanding *changes over time in the steady state*. Second, inflation also depends on the output gap, which should be understood as a catch-all for the economic analysis of the [ECB's] second pillar" (Gerlach 2004: 424).

Two major points of the Gerlach approach are worth mentioning:

(1) In any well defined macro theory, expectations should be explained in a model-consistent way, i.e. by solving the system of the expected values of the model's equations.<sup>4</sup> Contrary to that approach, Gerlach suppresses the internal determinants of the expected future rate of inflation and uses some measure of money growth instead. But why do market agents not build expectations according to the "true" model?

(2) As a corollary of this first objection, Gerlach offers no market transmission mechanism showing how money affects inflation, or any other macro variable, through factual market forces. One of Gerlach's critics stated: "If money does not affect objective behaviour of economic agents, it is hard to see how it would impact on their expectations of that behaviour. If money is to be integrated into two-pillar framework, it has to be done according to monetarist interpretation, based on the idea that money affects behaviour via liquidity constraints" (Frank Browne in Gerlach 2004: 429). Defending his approach, Gerlach (2004: 413) argues that "the assumption that money growth determines expected inflation should not be taken literally"; instead he interprets "the ECB as believing that money growth captures the stance of monetary policy and the general state of aggregate demand, and that it therefore can be used as a proxy for expected inflation".

This line of defence concedes that monetary policy acts through its impact on goods

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<sup>4</sup> Appendix I gives an example of how this can be done in a simple model.

demand. From this it follows that observing the output gap should be the main element of a monetary policy strategy; the monetary pillar appears superfluous, apart from "technical" problems.<sup>5</sup> Even authors who are classified as belonging to the "money camp" emphasize that "the quantity theory does *not* claim, and the importance claimed for monetary aggregates in the determination of inflation does *not* rest on, a direct channel linking money growth and inflation" (Nelson 2003: 1042). Nelson sharpens his point by arguing that providing econometric support for a direct money-inflation nexus over and above a link given by the output gap and the Phillips curve, suggests measurement errors or misspecification, but no additional evidence of the quantity theory. This line of reasoning paves the way for a "Keynesian" interpretation of a famous Friedman dictum: "In the long run, inflation is always and everywhere an excess nominal GDP phenomenon" (Gordon 1997: 17).

A possible way of arguing in favour of the importance of the monetary pillar can be found by making reference to the phenomenon of non-rational expectations on the part of market agents. The recently developed "learning" approach in macro theory has dissociated itself from the widely held dogma, stipulated in former times, that the assumption of model-consistent, rational expectations is an indispensable element of optimizing behaviour (Evans / Honkapohja 2001). Factual market mechanisms, particularly if dynamic and stochastic features prevail, simply may be too complex to be properly understood by individual agents (DeCanio 1979). They may suffer from incomplete knowledge with respect to the magnitude of functional parameters or even with respect to the qualitative character of market relations; moreover, it cannot be excluded a priori that they believe in "wrong" theories.

That is not to say that the money-inflation nexus is established by way of some superstitious belief.<sup>6</sup> There is broad empirical evidence of a cointegration of money and prices (Nelson 2003, De Grauwe / Polan 2005). But even if this finding does not necessarily verifies

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<sup>5</sup> The difficulty of gaining timely and non-distorted information from output gap figures may nevertheless justify the observation of more easy-to-collect money data, which may signal contemporary or imminent demand behaviour (Coenen et al. 2003, Galí et al. 2004: 19-20, Beck / Wieland 2006).

<sup>6</sup> If only monetarist *beliefs* produce inflation, this would be a case for welfare enhancing intervention on the science market: "It might be for the greater social good to tax all Monetarist writings and to subsidize Keynesian ones" (Hahn 1982: 93).



the quantity theory on a scientific level, market agents particularly in Europe may put some trust in the statement that exogenous money causes inflation. As long as this statement is debated within academic circles, with varying weights of the groups of supporters and critics, one cannot reject the hypothesis that a significant part of private agents subscribe to more or less elaborated versions of the traditional quantity theory.

### III A Phillips Curve Model with Endogenous Money

In order to facilitate the formal analysis, Gerlach's starting equation [1] is simplified by dropping the lagged inflation rate and by substituting actual for lagged output. This renders the equation conformable to the New Keynesian supply curve ( $\delta$  now represents a discount parameter which is near, but below unity); lagged macro variables will reappear anyway because of the specification of the forward expectation term.

$$p_t = \delta p_{t+1}^e + \alpha y_t + \varepsilon_t^s \quad [4]$$

Also, the lag structure of the nexus between inflation expectation and money growth trend is modified to

$$p_{t+1}^e = m_t^T \quad [5]$$

The general algorithm used by Gerlach to calculate future expected inflation is maintained. Substituting [5] and [3] into [4] yields

$$p_t = \delta \left[ \lambda m_t + (1-\lambda) m_{t-1}^T \right] + \alpha y_t + \varepsilon_t^s \quad [6]$$

Last period's version of [4] and [5] is given by

$$p_{t-1} = \delta m_{t-1}^T + \alpha y_{t-1} + \varepsilon_{t-1}^s \quad [7]$$

This can be solved for  $m_{t-1}^T$  and substituted into [6], which then yields

$$p_t = \delta \lambda m_t + (1-\lambda) p_{t-1} + \alpha y_t - \alpha (1-\lambda) y_{t-1} + \varepsilon_t^s - (1-\lambda) \varepsilon_{t-1}^s \quad [8]$$

The decision of explaining inflation expectations by the money growth trend instead of the growth rate itself ( $\lambda < 1$ ) leads to the inclusion of lagged terms in the Phillips curve equation.

Extending Gerlach's paper, a (traditional) demand curve is added that deviates from the currently wide-spread fashion of using a micro-founded Euler consumption demand function.<sup>7</sup> Therefore the forward-looking expected-income term does not appear in

$$y_t = g_t - \beta (i_t - p_{t+1}^e) \quad [9]$$

where  $g_t$  indicates autonomous spending (fiscal and private) and  $i_t$  is a short-term nominal interest rate that is controlled by the central bank. Here, also, future expected inflation is determined by [5] and [3]. This leads to

$$y_t = g_t - \beta [i_t - \lambda m_t - (1 - \lambda) m_{t-1}^T] \quad [10]$$

By using last period's version of [9]

$$y_{t-1} = g_{t-1} - \beta (i_{t-1} - m_{t-1}^T) \quad [11]$$

the demand function finally reads

$$y_t = (1 - \lambda) y_{t-1} + g_t - (1 - \lambda) g_{t-1} - \beta [i_t - (1 - \lambda) i_{t-1} - \lambda m_t] \quad [12]$$

A crucial deviation from the traditional quantity theory is the acknowledgement that money

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<sup>7</sup> The modern demand function, which for strange reasons is called New Keynesian, implies a strict determination of aggregate investment by household saving (Clarida et al. 1999). Gearing the macro demand equation to households' intertemporal consumption preferences alone downgrades investors to mere henchmen. This is hardly realistic. "The [rational expectations] model is inhabited by super-rational agents for whom the complexity of the world has few secrets. They continuously optimise their present and future consumption plans and are capable of calculating with great precision what the effects will be of interest changes implemented by the central bank. This is a fairytale world" (De Grauwe 2006). The modern demand function is built on the implicit assumption that perfect financial markets exist where information problems are absent and all agents can lend and borrow without any non-price constraints. "This assumption, of complete financial markets, lends itself admirably to the construction of soluble models with 'rigorous' micro-foundations of optimisation within a general equilibrium system. The problem, of course, is that the assumption has no connection with the real world" (Goodhart 2007: 19).

is endogenous; basically it adapts to the "needs of trade". This argument first was put forward as a critical point against Friedman's monetarism (Kaldor 1982), and it is now a standard theme that recently was emphasized in Woodford's (2006) assessment of the ECB's "monetary pillar" (although, the endogenous money concept in New Keynesian macroeconomics is not built on Kaldor's contribution). In order to capture "bubbles" in money demand the following money growth rate equation<sup>8</sup> is supplemented by a shock term which is modelled as independent from interest rates, firstly in order to facilitate the formal analysis, secondly because the sign of money demand reactions to interest rate movements may be ambiguous, particularly in constellations of a flat term structure.

$$m_t = p_t + y_t + \varepsilon_t^m \quad [13]$$

Surely, the knowledge of [13] is apt to undermine the monetarist belief [5]. But given the still rather high reputation of the quantity theory among economic experts and non-experts, and the necessity to explore the consequences of perhaps non-rational expectation formation, it appears sensible to include both equations [5] and [13] in the model setup.

Both types of shocks exhibit serial correlation and evolve as AR(1) processes where the persistence parameters are smaller than unity and the  $\omega_t$  terms represent white noise.

$$\begin{aligned} \varepsilon_t^m &= \eta^m \varepsilon_{t-1}^m + \omega_t^m \\ \varepsilon_t^s &= \eta^s \varepsilon_{t-1}^s + \omega_t^s \end{aligned} \quad [14]$$

Up to this point, money is a pure "bootstrap" variable in the macro process: apart from the shock term  $\varepsilon_t^m$ , it is determined by output and inflation, but it has only an "imagined" effect on inflation which becomes effective, though, through the forward expectation channel. A simple way of describing a factual *market* mechanism which justifies the monetarist belief [5] is to establish a spillover mechanism from money demand bubbles to goods demand. If the concurrence of excess money demand and asset price inflation is accepted as a stylized fact (it

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<sup>8</sup> Compared to the traditional quantity equation, in order to simplify the formal analysis, the growth rate of output is replaced by the output gap (and the logarithm of equilibrium output is normalized to zero).

was prevalent in the EMU area in recent years), a wealth effect might strengthen private consumption:

$$g_t = \bar{g} + \theta \varepsilon_t^m \quad [15]$$

Whereas, in a world of exogenous money, a money *supply* shock induces additional demand, here a money *demand* shock does not depress goods demand via a rising rate of interest because money is endogenous; rather, it produces a positive feedback loop on goods demand via some wealth effect.<sup>9</sup>

Turning to monetary policy, an obvious candidate for a "reasonable" and simple policy strategy is a rule<sup>10</sup> based on contemporary variables, which include, in an ECB style of policy making, deviations from a money growth target (or "reference value")  $m^*$  :

$$i_t = r_t^* + p_t + \gamma (p_t - p^*) + \varphi y_t + \mu (m_t - m^*) \quad [16]$$

The assumption of  $\mu > 0$ , of course, is highly contentious as it was found that the ECB hardly ever based its decisions on interest rate movements on deviations of money growth from the reference value (Gerlach 2004, Reichlin 2006). But  $\mu > 0$  in [16] can be justified on the ground that the consequences of money-based interest rate policies (which are recommended to the ECB time and again) have to be explored, particularly if market agents believe in some money-inflation nexus.

Inserting endogenous money  $m_t$  from [13] into [16], the hybrid Taylor rule

$$i_t = r_t^* + (1 + \gamma + \mu) p_t + (\varphi + \mu) y_t + \mu \varepsilon_t^m \quad [17]$$

(where the inflation target  $p^*$  and consequently the reference value of money growth  $m^*$  are

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<sup>9</sup> A much more elaborated way of getting money growth into a goods demand function would be the use of a non-separable utility function with money, as in Nelson (2002).

<sup>10</sup> Talking about "rules" is not meant to attribute a mechanical behaviour to central bankers. Interest rate reaction rules are regarded as lists of factors that guide monetary policy decisions. Assuming constant reaction parameters only helps to obtain a rough image of the ensuing effects. In the following model computations these parameters are varied, and some results hint to the merits of choosing optimal values of reaction parameters.

chosen to be zero) reveals that the inclusion of the money growth target has two effects:

- It merely amplifies the interest rate reactions to the inflation and the output gap, thereby establishing a reaction to the latter, if the policy setup otherwise would have ignored this issue (i.e. if  $\varphi = 0$ );
- and it gives rise to an interest rate response to money demand shocks or bubbles, which might help to stabilize financial markets and to dampen goods demand shocks which, by way of [15], result from those bubbles.

Equations [8], [12-15] and [17] can be reduced to a dynamic system where  $\Psi$ ,  $\Phi$  and  $\Theta$  represent  $2 \times 2$  matrices. The real equilibrium interest rate  $r_t^*$  is assumed to neutralize the constant part of autonomous spending  $\bar{g}$ , so that, apart from shocks, in equilibrium  $y_t = 0$ .

$$\begin{bmatrix} y_t \\ p_t \end{bmatrix} = \Psi \begin{bmatrix} y_{t-1} \\ p_{t-1} \end{bmatrix} + \Phi \begin{bmatrix} \varepsilon_{t-1}^m \\ \varepsilon_{t-1}^s \end{bmatrix} + \Theta \begin{bmatrix} \omega_t^m \\ \omega_t^s \end{bmatrix} \quad [18]$$

Dynamic stability depends on the eigenvalues of  $\Psi$  being smaller than unity in absolute terms. It turns out that this condition is met if the twofold inequality

$$\lambda < 1$$

$$(\alpha + \delta)\gamma + (1 - \delta)\varphi + (1 + \alpha)\mu > \frac{(1 - \delta)(\beta - 1)}{\beta} \quad [19]$$

holds. Obviously  $\mu > 0$  is not necessary for convergence. The problem can be stated more clearly, if the discount parameter, which is near unity anyhow, is neglected by letting  $\delta = 1$ . Then the second relation in [19] is simply  $\gamma + \mu > 0$ . This implies that the standard Taylor principle in dynamic macro models (Woodford 2001), a more than proportionate nominal-interest-rate reaction to inflation ( $\gamma > 0$ ), suffices to ensure that the equilibrium [ $p_t = p^*$ ,  $y_t = 0$ ] will be achieved, independently of any monetarist beliefs shared by market agents.

The different ability of interest rate policies to dampen the persistence of the market process during the adjustment after the occurrence of shocks, by reacting to inflation, output and money gaps respectively, can be assessed by differentiating the variable eigenvalue of  $\Psi$  (the other one is a constant) with respect to  $\gamma$ ,  $\varphi$  and  $\mu$ . It can be found that this eigenvalue is lowered

- if  $\gamma$  is increased,
- if  $\varphi$  is increased, given the case that  $\gamma + \mu < (1 - \delta)/\delta$ ,
- if  $\mu$  is increased, given the case that  $1 + \gamma < \varphi + 1/\beta + (1 + \alpha)/\delta$ .

These results indicate that a Taylor policy oriented by the inflation gap always helps to stabilize macro dynamics whereas output and money oriented interest rate policies do so only if some parameter restrictions are met.

A second test concerning the stabilization properties of the different varieties of Taylor interest rate policies explores the variance of the endogenous variables. From [18], a solution can be derived<sup>11</sup> that shows  $\text{var}(y)$  and  $\text{var}(p)$  as functions of  $\text{var}(\omega^m)$  and  $\text{var}(\omega^s)$ :

$$\begin{bmatrix} \text{var}(y) \\ \text{var}(p) \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \text{var}(\omega^m) \\ \text{var}(\omega^s) \end{bmatrix} \quad [20]$$

The coefficients  $\Lambda_{ij}$  represent (rather unwieldy) multipliers, containing all parameters of the model. Due to the complexity of the model, a numerical calibration of parameters is necessary. With regard to the slope of the supply function and the real-interest-rate elasticity of goods demand, estimations in the literature are in the range of  $\alpha = 0.024 \dots 0.3$  and  $\beta = 0.16 \dots 6.37$ , respectively (Evans / Honkapohja 2006). Throughout the paper we follow a middle course by selecting  $\alpha = 0.1$  and  $\beta = 3$ .<sup>12</sup> Furthermore, besides  $\delta = 0.99$ ,  $\lambda = 0.1$  was chosen which is roughly in line with Gerlach's calculation.

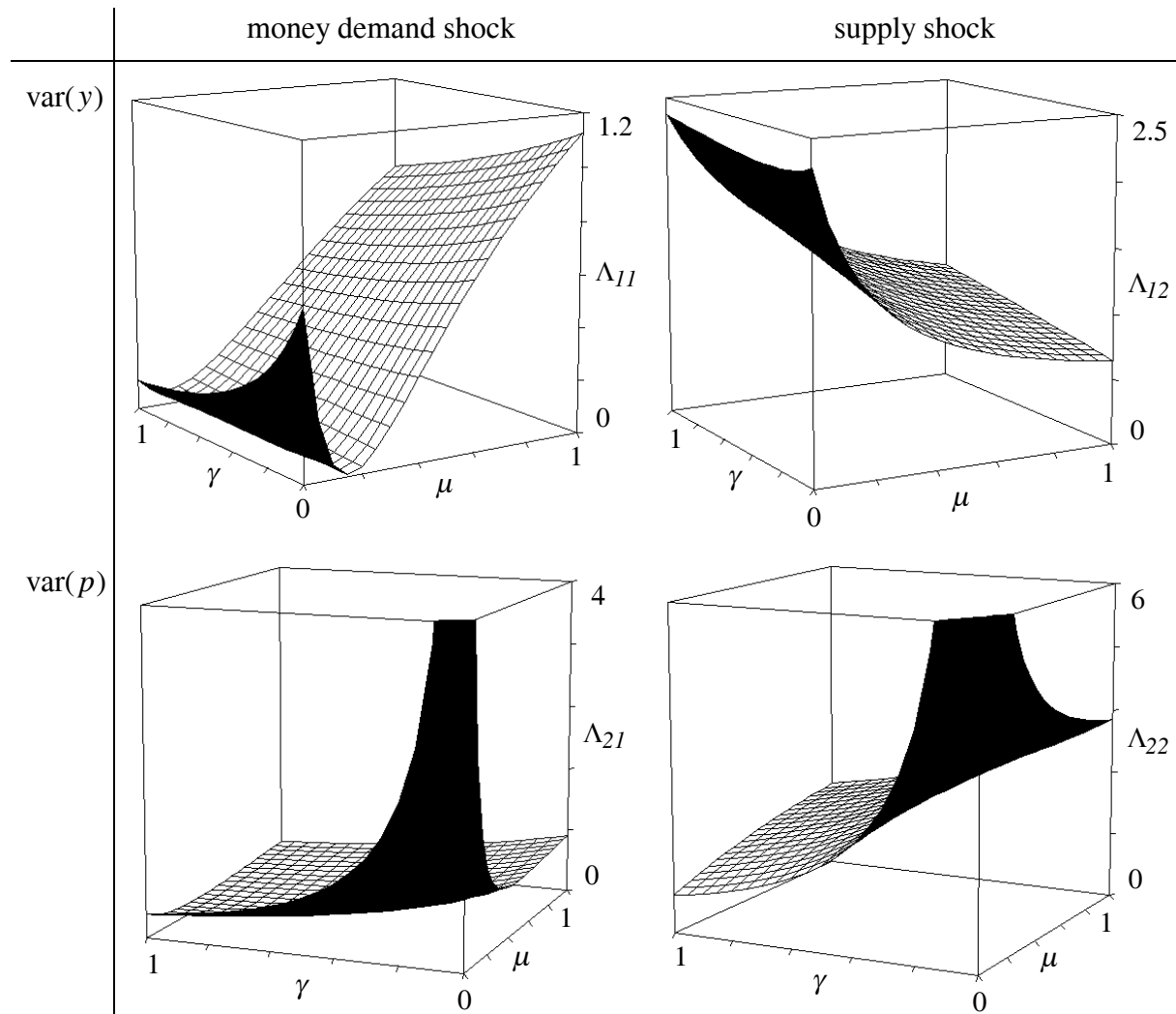
In order to demonstrate, firstly, the effects of varying Taylor coefficients that are attached to the inflation and money growth gap,  $\gamma$  and  $\mu$ , respectively, the other parameters were fixed at  $\varphi = 0.1$  and  $\eta^m = \eta^s = \theta = 0.5$ . The variance of money demand is normalized to unity, whereas the variance of supply shocks is chosen to be 0.1.<sup>13</sup> Figure 1 displays the dependence of the magnitude of the  $\Lambda_{ij}$  terms of Equation [20] on variations of  $\gamma$  and  $\mu$

<sup>11</sup> Details are given in Appendix II.

<sup>12</sup> A series of computations, including the values used by Woodford (2007),  $\alpha = 0.024$  and  $\beta = 6.25$ , showed that the results of the paper do not depend on this parameter choice.

<sup>13</sup> This differentiation was made in order to compensate for the relatively weak, indirect impact of money demand shocks in the model setup, as compared to supply shocks which have a direct bearing on the macro variables.

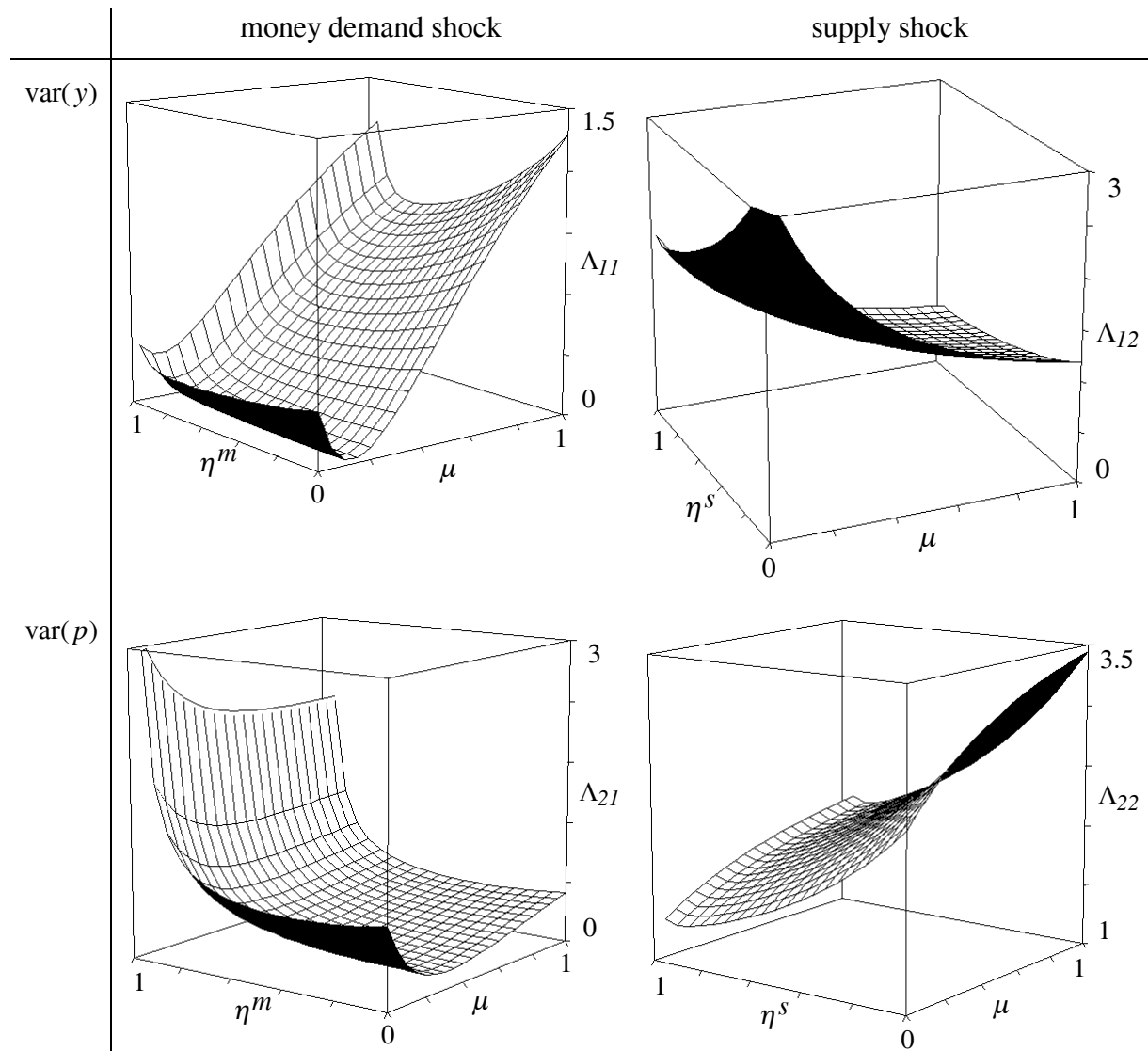
Figure 1 (illustrating Equation [20]): multipliers of shock variability on output and inflation variability, depending on reactions to the inflation and the money gap



between zero and unity.

- With regard to inflation variability, enlarging  $\gamma$  and  $\mu$  starting from low values lowers both the impact of supply and money demand shocks (cases  $\Lambda_{21}$  and  $\Lambda_{22}$ ); but given some moderate reaction to the money gap, inflation variability can be contained more efficiently by raising  $\gamma$ .
- A stronger reaction to the money growth gap also helps to dampen the impact of supply shocks on output variability (case  $\Lambda_{12}$ ), a service that  $\gamma$ , being oriented by the inflation issue, would not deliver; the coefficient  $\mu$  in [17] acts indirectly as a tool for stabilizing output around the full-employment level.
- If  $\gamma$  is low, money demand shocks have a non-linear effect on output variability (case  $\Lambda_{11}$ ), which results from the feedback from  $\varepsilon_t^m$  on autonomous spending. A rising value of  $\mu$

Figure 2 (illustrating Equation [20]): multipliers of shock variability on output and inflation variability, depending on shock persistence and the reaction to the money gap



at first succeeds to neutralize this induced goods demand shock, but beyond some optimal value a precautionary policy impulse that responds to the building up of money balances puts an unnecessary restriction on demand. With  $\theta = 0$  in [15], this ambiguity would be absent. The slope of the plane would be positive throughout, indicating a clear welfare loss due to the interest rate reaction to excess money growth. If *optimal* interest rate policies would be implemented instead of instrument rules, the central banker ought to have precise information about the characteristics of the various shocks; optimal reaction parameters would be difficult to compute in reality. Obviously, keeping  $\mu$  at zero and  $\gamma$  high, yields an efficient and more robust response to the impact of money demand shocks on output variability.



All these results imply that reacting to the money growth gap ( $\mu > 0$ ) delivers questionable stabilization results. It produces unnecessary output volatility in case of money demand shocks, whereas the inflation effects of these shocks can be handled more efficiently by using the inflation-gap reaction coefficient  $\gamma$ . The same is true with regard to the impact of supply shock effects of inflation: stabilization can also be obtained by relying on  $\gamma$ . The remaining case,  $\Lambda_{12}$ , only at first glance speaks for  $\mu > 0$ ; but the interest rate rule [17] shows that output can be protected against supply shocks also by increasing the value of  $\varphi$ .

*Figure 2* explores the consequences of pursuing (also) a money target further by laying the focus on variable persistence of shocks. Now, the inflation reaction parameter is fixed at  $\gamma = 0.5$ , whereas  $\mu$ ,  $\eta^m$  and  $\eta^s$  are flexible, and the other parameters are kept unchanged at their values given above. In all cases, variability of endogenous variables explodes if  $\eta^m$  and  $\eta^s$  approach unity.

- Again, there appears to be an optimal degree of the money gap parameter in the case of money demand shocks ( $\Lambda_{11}$  and  $\Lambda_{21}$ ). The macroeconomic costs of exceeding this optimal value of  $\mu$  are particularly pronounced with regard to output. A larger reaction parameter  $\mu$  does not dampen output or inflation variability if the persistence of money demand shocks increases.
- More persistent supply shocks reduce the variance of inflation anyhow (case  $\Lambda_{22}$ ); if supply shocks are more rigid, inflation fluctuates less. A rising value of  $\mu$  helps to keep output variability low in case of supply shocks (case  $\Lambda_{12}$ ), but so would  $\varphi$  also, which is not shown in a simulation, but can be seen from an inspection of [17].

#### **IV A Model with Hybrid Inflation Expectations and Learning**

A major shortcoming of the above analysis is that model-consistent expectations have been completely suppressed in favour of the supposed belief in the inflationary impact of money growth. Therefore an obvious modification of the model is to allow "rational" expectations in addition to "monetarist" beliefs. This is achieved by distinguishing between two groups of private agents who – by analogy to approaches used in exchange rate theory (De Grauwe /

Grimaldi 2002) – are characterized by different habits of expectation formation.

The starting point is the supply function where expected future inflation now is determined, partly, by the influence of trend money growth and, partly, by model-consistent forward-looking expectations (which are to be explained endogenously). The term  $p_{t+1}^e$  in [21] represents these rational forward-looking expectations only, whereas predicted inflation by monetarist agents is given directly by  $m_t^T$ . The weights attached to both factors add up to unity.

$$p_t = \delta \left[ \sigma m_t^T + (1 - \sigma) p_{t+1}^e \right] + \alpha y_t + \varepsilon_t^s \quad [21]$$

Substituting [3] into [21] and proceeding by analogy to the steps from [6] to [8], the supply function evolves into

$$\begin{aligned} p_t = & (1 - \sigma) \delta \left[ p_{t+1}^e - (1 - \lambda) p_t^e \right] + \sigma \delta \lambda m_t + (1 - \lambda) p_{t-1} \\ & + \alpha \left[ y_t - (1 - \lambda) y_{t-1} \right] + \varepsilon_t^s - (1 - \lambda) \varepsilon_{t-1}^s \end{aligned} \quad [22]$$

By following the same pattern of derivation, the demand function

$$y_t = g_t - \beta \left[ i_t - \sigma m_t^T - (1 - \sigma) p_{t+1}^e \right] \quad [23]$$

turns out to be

$$\begin{aligned} y_t = & (1 - \lambda) y_{t-1} + g_t - (1 - \lambda) g_{t-1} \\ & - \beta \left[ i_t - (1 - \lambda) i_{t-1} - (1 - \sigma) p_{t+1}^e + (1 - \sigma) (1 - \lambda) p_t^e - \sigma \lambda m_t \right] \end{aligned} \quad [24]$$

Taking [13-15], [17], [22] and [24], the model economy can be written in the compact form

$$\mathbf{v}_t = \mathbf{\Omega} \mathbf{v}_{t+1}^e + \mathbf{\Psi} \mathbf{v}_{t-1} + \mathbf{\Phi} \mathbf{s}_{t-1} + \mathbf{\Theta} \mathbf{e}_t \quad [25]$$

Here,  $\mathbf{\Omega}$ ,  $\mathbf{\Psi}$ ,  $\mathbf{\Phi}$  and  $\mathbf{\Theta}$  are  $2 \times 2$  matrices.<sup>14</sup> The endogenous variables are comprised in the

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<sup>14</sup> They are different from the equally named matrices in [18].

vector  $\mathbf{v}_t = [y_t; p_t]'$ , their next period's expected values in  $\mathbf{v}_{t+1}^e$ , and their lagged values in  $\mathbf{v}_{t-1}$ . It is assumed that agents can observe time  $t$  shocks in the current period; therefore  $p_t^e = p_t$ . The equation is completed by  $\mathbf{s}_{t-1} = [\varepsilon_{t-1}^m; \varepsilon_{t-1}^s]'$  and  $\mathbf{e}_t = [\omega_t^m; \omega_t^s]'$ . As in the model of Section III,  $r_t^* = \bar{g}/\beta$  and  $p^* = m^* = 0$ . In order to simplify the calculation, the persistence parameters are assumed to be equal ( $\eta^m = \eta^s = \eta$ ).

The model architecture reflects the existence of two groups of agents, from which one adheres to monetarist thinking whereas the other builds expectations according to rational principles. The distribution of knowledge among market agents and policymakers is as follows:

- Monetarist believers, as in Section III above, build forward inflation expectations by observing the money growth trend. They do not know, or do not take into account, that money is endogenous and that there are other agents who apply a more elaborate concept of predicting inflation.
- This second group of agents understands the basic logic of the macro system, including the existence of "stubborn" disciples of the quantity theory. As a consequence, rational agents also attribute inflationary expectations to a rising money stock, just because they know that inflationary pressures arise from the beliefs of monetarist agents. The prediction of future inflation is derived from the preceding equations (see below). It is assumed that this second group does not suffer from model, but from parameter uncertainty.
- The central bank follows the instructions given in the Taylor rule [17]. As the rule contains only contemporary variables, there is no need to decide on the most efficient pattern of making inflation predictions. Central bankers subscribe to an eclectic and pragmatic way of policy making; they include in their reaction rule all variables that might be relevant for the issue of monetary stabilization: the output gap, money growth and inflation itself.

The group of rational-expectation believers understands that the market process [25], which depends on their own prediction of future inflation, in equilibrium, including the expectation effect, will show the form

$$\mathbf{v}_t = \mathbf{A} \mathbf{v}_{t-1} + \mathbf{B} \mathbf{s}_{t-1} + \mathbf{C} \mathbf{e}_t \quad [26]$$

where the matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  contain coefficients that are not given by the true parameters of the market process [25].<sup>15</sup> Rather, they represent a preliminary estimation on behalf of rational, but imperfectly informed market agents. Based on [26], which represents the "perceived law of motion" (PLM), market agents build their expectations on next period's variables:

$$\mathbf{v}_{t+1}^e = \mathbf{A} \mathbf{v}_t^e + \mathbf{B} \mathbf{s}_t^e = \mathbf{A} (\mathbf{A} \mathbf{v}_{t-1} + \mathbf{B} \mathbf{s}_{t-1} + \mathbf{C} \mathbf{e}_t) + \mathbf{B} (\eta \mathbf{s}_{t-1} + \mathbf{e}_t) \quad [27]$$

These expectations then become a part of the "actual law of motion" (ALM). Inserting [27] into [25] yields, after rearranging terms,

$$\mathbf{v}_t = (\mathbf{\Omega} \mathbf{A}^2 + \mathbf{\Psi}) \mathbf{v}_{t-1} + (\mathbf{\Omega} \mathbf{A} \mathbf{B} + \mathbf{\Omega} \mathbf{B} \eta + \mathbf{\Phi}) \mathbf{s}_{t-1} + (\mathbf{\Omega} \mathbf{A} \mathbf{C} + \mathbf{\Omega} \mathbf{B} + \mathbf{\Theta}) \mathbf{e}_t \quad [28]$$

The comparison of [26] and [28] now allows of a learning process: market agents are supposed to adjust their preliminary chosen parameter values if actual experience deviates from these figures (Evans / Honkapohja 2001). In the most simple form, the adjustment process takes the form

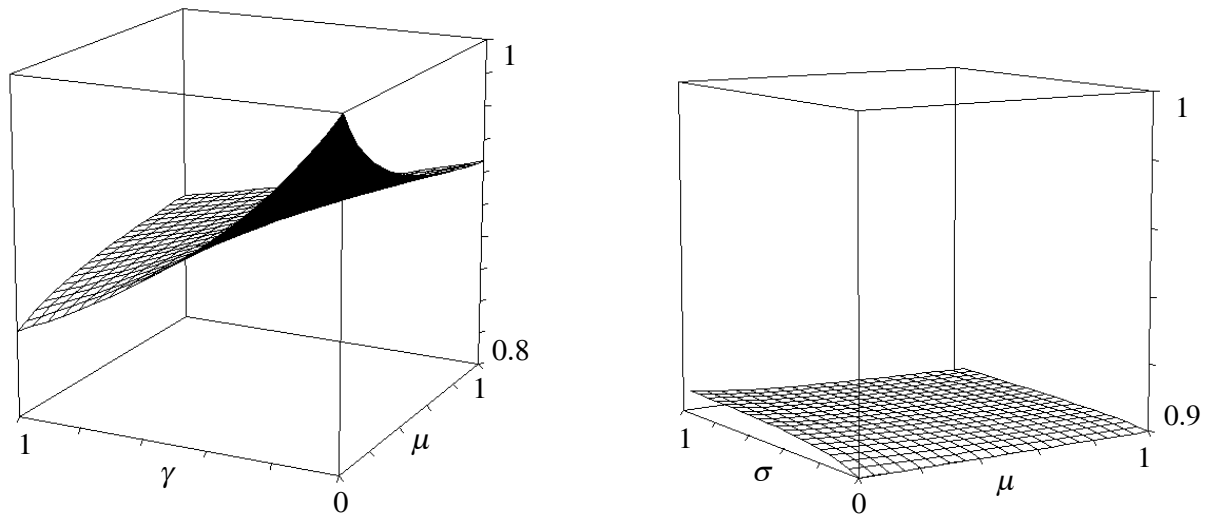
$$\begin{aligned} \dot{\mathbf{A}} &= (\mathbf{\Omega} \mathbf{A}^2 + \mathbf{\Psi}) - \mathbf{A} \\ \dot{\mathbf{B}} &= (\mathbf{\Omega} \mathbf{A} \mathbf{B} + \mathbf{\Omega} \mathbf{B} \eta + \mathbf{\Phi}) - \mathbf{B} \\ \dot{\mathbf{C}} &= (\mathbf{\Omega} \mathbf{A} \mathbf{C} + \mathbf{\Omega} \mathbf{B} + \mathbf{\Theta}) - \mathbf{C} \end{aligned} \quad [29]$$

Equilibrium is reached if assumed and realized values conform. Then the solution for the matrices  $\mathbf{A}^*$ ,  $\mathbf{B}^*$  and  $\mathbf{C}^*$ , in principle, can be computed from

$$\begin{aligned} \mathbf{\Omega} \mathbf{A}^2 + \mathbf{\Psi} &= \mathbf{A} \\ \mathbf{\Omega} \mathbf{A} \mathbf{B} + \mathbf{\Omega} \mathbf{B} \eta + \mathbf{\Phi} &= \mathbf{B} \\ \mathbf{\Omega} \mathbf{A} \mathbf{C} + \mathbf{\Omega} \mathbf{B} + \mathbf{\Theta} &= \mathbf{C} \end{aligned} \quad [30]$$

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<sup>15</sup> The following solution algorithm extends the simple example given in Appendix I. It applies the techniques described in Evans / Honkapohja (2001), Bullard / Mitra (2002) and McCallum (2003). Equation [26] represents the "Minimal State Variable" (MSV) solution (McCallum 1983), which uses the smallest number of state variables that conform to the structural model [25].

Figure 3: eigenvalues of  $\mathbf{A}^*$  with fixed  $\sigma = 0.5$  (left), and  $\gamma = 0.5$  (right)

If  $\mathbf{A}^*$  is found from the first equation<sup>16</sup>, the second and third equation can be solved for  $\mathbf{B}^*$  and  $\mathbf{C}^*$ . This completes the general solution given in [26].

The question whether the adjustment process of parameters actually converges to the solution set  $[\mathbf{A}^*, \mathbf{B}^*, \mathbf{C}^*]$  is answered by checking the dynamic stability of the system [29]. Applying the test criteria given in Evans / Honkapohja (2001: 237-8), it can be shown by way of numerical calculations that "expectational stability" of the learning process is given if only one of the Taylor equation coefficients  $\gamma$  or  $\mu$  is positive.

Next, the characteristics of the solution with regard to convergence and persistence can be demonstrated by analyzing the variable eigenvalue of the  $\mathbf{A}^*$  matrix (the second one is a constant). The following results stand out:

- If the weight of the monetarist group exceeds some small limit, convergence cannot be obtained by only stabilizing the output gap ( $\varphi > 0$  with  $\gamma = \mu = 0$ ).
- If the relative weight of the two groups of agents is fixed at  $\sigma = 0.5$ , the (left) graph of

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<sup>16</sup> Technical difficulties arise from the matrix quadratic in the first equation. It was solved, after a transformation into  $(\mathbf{I} - \Omega \mathbf{A}) \mathbf{A} = \Psi$ , where  $\mathbf{I}$  indicates the unity matrix, by multiplying and adding up the elements on the left hand side, so that a  $2 \times 2$  matrix emerges on both sides. Equating term by term on both sides then gives four (non-linear) equations which determine the four coefficients of  $\mathbf{A}^*$ . Two possible solutions emerge. According to the logic of MSV solutions, the set containing smaller values was chosen (where the eigenvalues are smaller than unity); this widely shared analytical convention is derived from the supposition that market agents, when designing their PLM, in principle envisage a stable process (McCallum 2003).

Figure 3 replicates the analytical finding that either  $\gamma$  or  $\mu$  ought to be positive. Increasing  $\gamma$  lowers persistence throughout, whereas  $\mu$  does so only with given small values of  $\gamma$ .

- If the inflation gap coefficient is fixed at  $\gamma = 0.5$ , the (right) graph shows that increasing the share<sup>17</sup> of monetarist believers adds (slightly) to persistence; but this effect can be cushioned by strengthening the weight given to the money growth target in the Taylor rule.<sup>18</sup>

By analogy to the procedure in Section III<sup>19</sup> the variability of output and inflation can be computed as

$$\begin{bmatrix} \text{var}(y) \\ \text{var}(p) \end{bmatrix} = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \begin{bmatrix} \text{var}(\omega^m) \\ \text{var}(\omega^s) \end{bmatrix} \quad [31]$$

The matrix coefficients are displayed in *Figures 4* and *5*. Firstly, the specification  $\varphi = 0.1$  and  $\gamma = \eta = \theta = 0.5$  was made in order to concentrate on the effects of monetarist beliefs ( $\sigma$ ) and monetarist policies ( $\mu$ ); as in Section III, the normalization  $\text{var}(\omega^m) = 1$  and  $\text{var}(\omega^s) = 0.1$  is used.<sup>20</sup>

- The impact of money demand shocks on output can be neutralized by a cautiously chosen reaction to money growth target violations. As in the scenarios of *Figures 1* and *2*, this beneficial effect is reversed if the interest rate reaction is increased beyond the optimal degree. The weight of the group of monetarists among market agents is more or less irrelevant for the magnitude of  $\Delta_{11}$ .
- Obviously, money demand shocks produces some extra inflation by way of inflationary expectations. The multiplier  $\Delta_{21}$  increases with the share of monetarist believers. Again, there appears to be an optimal policy reaction  $\mu$ , particularly if  $\sigma$  is high. But too high values of  $\mu$  add to inflation variability.
- The effect of supply shocks on output is dampened if monetarist beliefs and/or monetarist

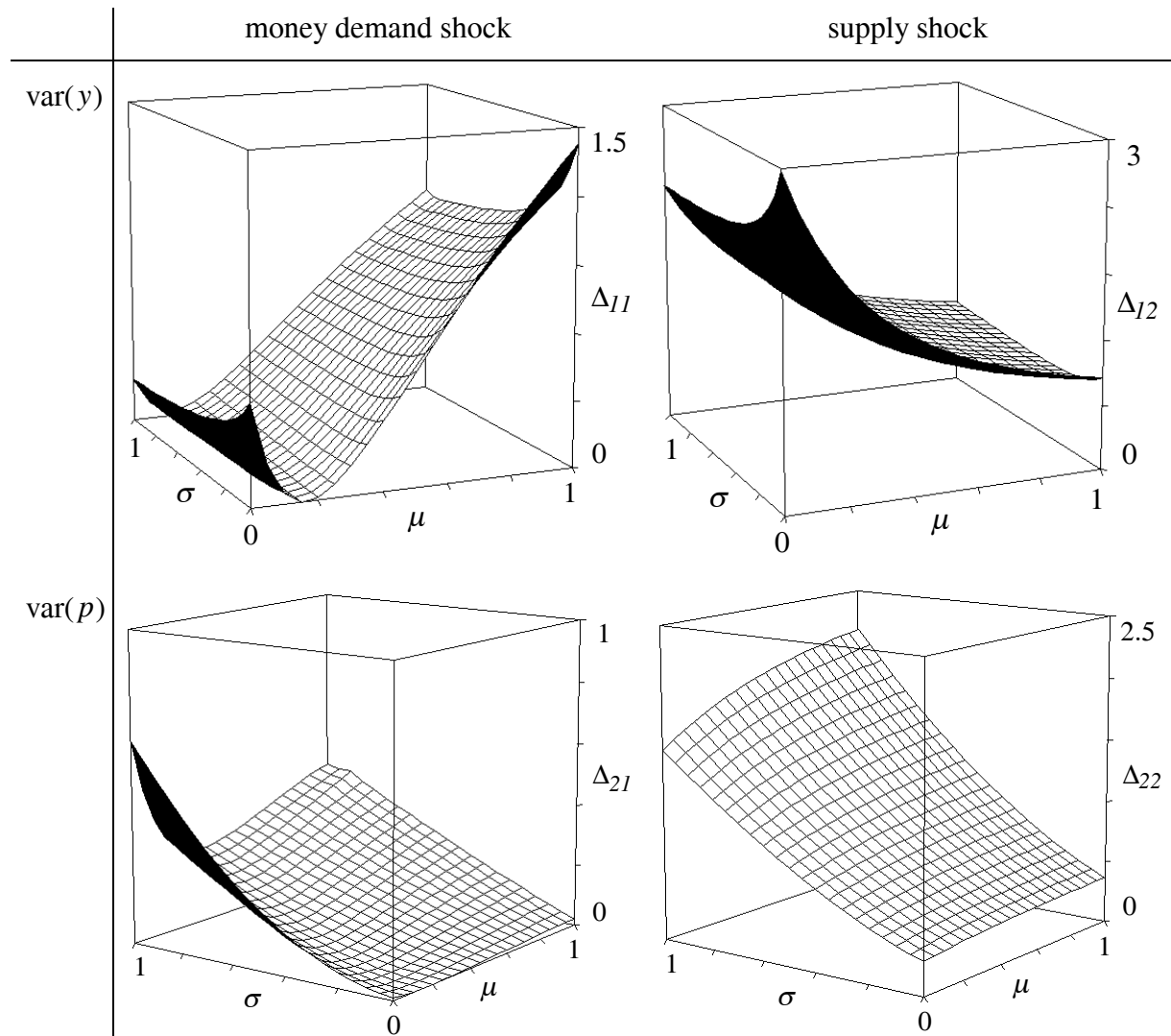
<sup>17</sup> Note that the model's solution is not defined at the limiting point  $\sigma = 1$ . The graphs therefore leave out that region.

<sup>18</sup> Both graphs assume a weak interest rate reaction to the output gap ( $\varphi = 0.1$ ).

<sup>19</sup> Details in Appendix II.

<sup>20</sup> The equations are not defined at the limiting points  $\sigma = 0$  and  $\sigma = 1$ . Therefore, the graphs leave out these regions.

Figure 4 (illustrating Equation [31]): multipliers of shock variability on output and inflation variability, depending on the share of monetarist believers and the reaction to the money gap

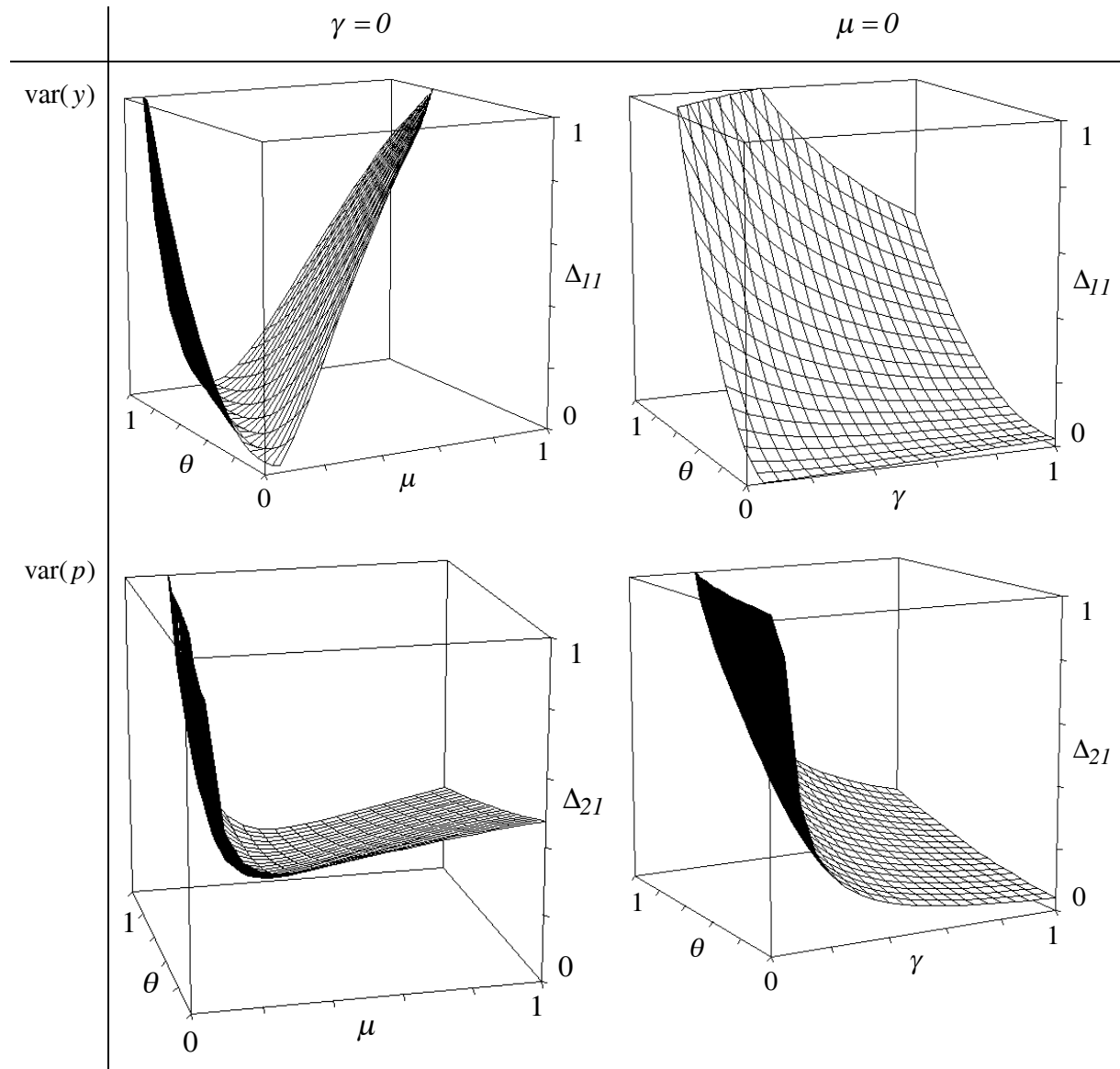


policies exist (case  $\Delta_{12}$ ). However, monetarist beliefs multiply the impact of these shocks on inflation (case  $\Delta_{22}$ ), again because money growth is endogenously increased, which then in turn adds to inflationary expectations.

- In general, an increasing weight of monetarist beliefs adds to the variability of inflation. Except for case  $\Delta_{12}$ , strong monetarist policies increase output and inflation volatility; positive money demand shocks prompt the central bank to react by rising interest rates that in turn depress demand and (to a weaker extent) push the rate of inflation from its target.

Finally, in *Figure 5*, the focus is on the variable goods demand effect that is caused by a money demand shock (the supply shock is no longer discussed). The question is whether a rising value of  $\theta$  in  $g_t = \bar{g} + \theta \varepsilon_t^m$  is dampened more efficiently by interest rate reactions to

Figure 5 (illustrating Equation [31]): multipliers of money demand shock variability on output and inflation variability, depending on a variable goods demand effect of money demand shocks, the reaction to the money gap and the reaction to the inflation gap



the money or the inflation gap, respectively. Hence, in addition to  $\varphi = 0.1$ ,  $\sigma = \eta = 0.5$  was chosen, and the  $\theta$  effect is displayed vis-à-vis variable values of  $\mu$  and  $\gamma$ .

- The variable  $\theta$  effect emphasizes what has been found above: there might be an efficient response by a money gap reaction of interest rates (if the inflation gap coefficient is zero). A positive relation  $\mu = f(\theta)$  is particularly pronounced with regard to output (case  $\Delta_{11}$ ). Again, the graphs also show the risks of money targeting; output variability increases markedly with wrongly chosen values of  $\mu$ .
- If on the other hand the central bank refrains from money targeting ( $\mu = 0$ ), the graphs



show that a stronger goods demand effect of money demand shocks can unambiguously be countered by raising the inflation gap coefficient. It can be shown (no graph given) that using the output gap coefficient  $\varphi$  instead of  $\gamma$  would yield a very similar result with respect to output variability. However, this policy strategy would not succeed to dampen the effect of a larger  $\theta$  on inflation variability.

## V Conclusions

The current debate on monetary policy strategies in the euro area time and again runs into the question of attributing an appropriate role to monetary variables. After the demise of the IS-LM model, it was difficult to find a place for money in an integrated formal theory of the transmission process. The two-pillar Phillips curve offers a framework that allows to identify empirically long-run (low-frequency) movements of inflation as distinct from short-run (high-frequency) changes of inflation; the former, i.e. the shift parameter of the Phillips curve can be assigned to the money growth trend, and the latter to the output gap.

From the point of view of macroeconomic theory, the two-pillar Phillips curve implies that money growth affects inflation only by way of inflationary expectations. This deficiency can be redressed if the two-pillar supply curve is used as a component of a simple macro model where the behaviour of the central bank is described by a Taylor rule. Money growth basically is endogenous, depending on output and inflation, but can also form demand-side determined bubbles. The model takes into account serially correlated supply and money demand shocks, and allows for goods demand shocks that result from these money demand bubbles. Surely, the money growth trend is regarded as an indicator for long-term inflationary risks; however, the notion "long-term" does not mean that something might happen in the far future, but rather, that it will happen instantaneously as a by-product of a return to a state of equilibrium. Hence, the belief in a fairly stable money demand function implies a high probability of a positive goods demand shock in times of excess money growth.

This poses the question of an adequate policy response. Adding a money growth target to a standard Taylor rule essentially has the effect of strengthening interest rate reactions to the

inflation and the output gap. The crucial point is the occurrence of money demand shocks that may trigger an ensuing goods demand shock. The current paper has analyzed the consequences of variable interest rate reactions to excess money growth. The model simulation shows that exact quantitative information on the characteristics and the interdependence of money-demand and goods-demand shocks are required on the part of the central bank. Otherwise interest rate reactions to money gaps cannot achieve efficient stabilization results.

In case of imperfect information, relying on the more traditional inflation gap policy generally appears to be the more robust monetary policy strategy (in some cases, putting additional emphasis on the output gap coefficient also promises to deliver good results). If – contrary to the model setup in this paper – there is only some *probability* of a goods demand effect of a money demand bubble, there is even less reason to pursue a prophylactic monetary restriction in case of excess money growth. Of course, no serious critic of the monetary pillar ever demanded that the ECB should give up to observe monetary data. This paper reaches – from a somewhat different perspective – the same result as the recent work of Woodford (2006, 2007): even if a two-pillar Phillips curve holds up as an empirical phenomenon, this does not justify a two-pillar monetary policy strategy.

The existence of monetarist beliefs among market agents aggravates the problem of monetary stabilization as these beliefs produce inflationary expectations, i.e. supply side pressure on price formation, which inevitably poses a conflict for any central bank. A pragmatic response consists of fighting inflation and output gaps when they emerge; public debates on the necessity of controlling money gaps exacerbate the problem that money gap control is meant to solve.

*Appendix I: building model-consistent rational expectations*

As an example, the following system of inflation  $p_t$  and the output gap  $y_t$ , which for simplicity is designed as AR(1) with  $\phi < 1$  and a demand shock  $\varepsilon_t^d$ ,

$$\begin{aligned} p_t &= \delta p_{t+1}^e + \alpha y_t + \varepsilon_t^s \\ y_t &= \phi y_{t-1} + \varepsilon_t^d \end{aligned} \quad [\text{A.1}]$$

has a general solution of the form

$$p_t = c y_t + \varepsilon_t^s \quad [\text{A.2}]$$

where  $c$  is an undetermined coefficient. It follows that, taking time  $t$  expectations,

$$p_{t+1}^e = c y_{t+1}^e = c \phi y_t \quad [\text{A.3}]$$

After inserting [A.3] into [A.1], and then comparing the coefficients of [A.1] and [A.3], we find the specific solution

$$p_t = \frac{\alpha}{1 - \delta \phi} y_t + \varepsilon_t^s \quad [\text{A.4}]$$

*Appendix II: computing variances of endogenous variables*

System [20] is transformed into two equations for output and inflation, respectively, where the matrix coefficients of  $\mathbf{X} = \Psi, \Phi, \Theta$  are denoted by  $X_{ij}$  terms. Squaring both sides of each equation gives

$$\begin{aligned} \text{var}(y) &= \frac{\Psi_{12}^2 \text{var}(p) + \Phi_{11}^2 \text{var}(\varepsilon^m) + \Phi_{12}^2 \text{var}(\varepsilon^s) + \Theta_{11}^2 \text{var}(\omega^m) + \Theta_{12}^2 \text{var}(\omega^s)}{1 - \Psi_{11}^2} \\ \text{var}(p) &= \frac{\Psi_{21}^2 \text{var}(y) + \Phi_{21}^2 \text{var}(\varepsilon^m) + \Phi_{22}^2 \text{var}(\varepsilon^s) + \Theta_{21}^2 \text{var}(\omega^m) + \Theta_{22}^2 \text{var}(\omega^s)}{1 - \Psi_{22}^2} \end{aligned} \quad [\text{A.5}]$$

Taking into account that [14] implies

$$\text{var}(\varepsilon^m) = \frac{\text{var}(\omega^m)}{1 - (\eta^m)^2} \quad \text{and} \quad \text{var}(\varepsilon^s) = \frac{\text{var}(\omega^s)}{1 - (\eta^s)^2} \quad [\text{A.6}]$$

yields the matrix equation [20].

The same procedure is used for computing the expression [31]. We start from [26], substitute the set of solutions  $\mathbf{A}^*, \mathbf{B}^*, \mathbf{C}^*$ , and obtain

$$\begin{aligned} \text{var}(y) &= \frac{\left(A_{12}^*\right)^2 \text{var}(p) + \left(B_{11}^*\right)^2 \text{var}\left(\varepsilon^m\right) + \left(B_{12}^*\right)^2 \text{var}\left(\varepsilon^s\right) + \left(C_{11}^*\right)^2 \text{var}\left(\omega^m\right) + \left(C_{12}^*\right)^2 \text{var}\left(\omega^s\right)}{I - \left(A_{11}^*\right)^2} \\ \text{var}(p) &= \frac{\left(A_{21}^*\right)^2 \text{var}(y) + \left(B_{21}^*\right)^2 \text{var}\left(\varepsilon^m\right) + \left(B_{22}^*\right)^2 \text{var}\left(\varepsilon^s\right) + \left(C_{21}^*\right)^2 \text{var}\left(\omega^m\right) + \left(C_{22}^*\right)^2 \text{var}\left(\omega^s\right)}{I - \left(A_{22}^*\right)^2} \end{aligned} \quad [\text{A.7}]$$

By using again [A.6], the equations in [A.7] can be solved for  $\text{var}(y)$  and  $\text{var}(p)$ .

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