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Abstract
Is the Euro area as a whole, or are individual Euro-area member countries facing a period of sustained lower economic growth, a phenomenon known as secular stagnation? We tackle this question by estimating equilibrium real interest rates and comparing them to actual real rates. Since the financial crisis has altered the degree of leverage in several European economies, we expand our model to incorporate the financial cycle. We estimate the model for the Euro area as a whole and for nine Euro-area member countries. Incorporating the financial cycle changes the estimated equilibrium real interest rates: For some of Euro-area member countries, estimates of the equilibrium real interest rate are substantially higher than the standard estimates. In other cases, including our estimates for the Euro area as a whole, the estimated equilibrium real rates are slightly lower than without taking the financial cycle into account but are still higher than the actual rates. This indicates that real monetary policy rates were set even more systematically and consistently below (or not as far above) the natural real rate. Comparing the sequence of actual and equilibrium real rates, only Belgium, France, and Greece are likely to face a period of secular stagnation.

JEL-Classification: E43, F45, C32

Keywords: equilibrium real interest rate, Euro area, financial cycle, heterogeneity, monetary policy, secular stagnation
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Brussels, Essen and Giessen

February 2018

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1. Introduction

The financial crisis has affected European economies in a variety of ways. Perhaps the most important change can be seen in the altered growth dynamics in Europe as well as other industrialized countries. Summers (2014a, 2014b, 2014c) views this low-growth era as a new period of secular stagnation and thus as a structural and non-temporary phenomenon.

Is secular stagnation really a problem in the Euro area as a whole or in the individual member states? The question is necessarily an empirical one. We tackle it in this paper through a comparison of the real interest rate and the unobservable equilibrium real interest rate. If the latter is too low for the former to be reached, secular stagnation may occur. However, since the equilibrium real interest rate is an unobservable variable, it must be estimated. We employ the most frequently used model to estimate this variable, the Laubach-Williams model (Laubach and Williams, 2003).

But the standard Laubach-Williams model does not explicitly model important parts of the economy that have changed over the course of the financial crisis. The most important missing determinant is obviously the financial cycle, as proxied, for instance, by the public and private debt level, which in turn should have an influence on the equilibrium real rate (Claessens et al., 2011, Drehmann et al., 2012, Rey, 2015, and Stremmel, 2015). To identify the specific phases of the financial cycle, the so-called leverage gap and the debt-to-service ratio are frequently used indicators. The leverage gap is usually modelled by summing credit to households and non-financial corporations and dividing it by the non-financial assets of the same two groups (Juselius et al., 2016). The debt-to-service ratio denotes the ratio of cash available for debt

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3 See, e.g., Borio et al. 2017 upon this issue.
servicing to interest, principal, and lease payments, but the respective time series is often not available for longer time spans.

Therefore, we expand the model by adding (the phases of) the financial cycle as Juselius et al. (2016) did for the US. Augmenting the model in this way has the potential to change the equilibrium real rate considerably and thus to change our inferences as to whether or not countries face secular stagnation. If our estimates of the equilibrium real interest rates based on the financial cycle-augmented models turn out to be higher than the standard ones without incorporating the financial cycle, we feel justified in concluding that real monetary policy rates were set even more systematically and consistently below (or not as far above) the natural real rate. In other words, this would prove that attributing the decline in real interest rates primarily to an exogenously caused reduction in the natural interest rate and not taking financial factors into account leads to erroneous conclusions (Borio, 2016).

We thus estimate a standard Laubach-Williams model and a financial cycle-augmented version thereof, and compare both to the actual real rate in order to infer whether a country faces secular stagnation. We do so for the Euro area as a whole and for nine individual member countries.

The remainder of the paper is organized as follows. In Section 2 we present the theoretical basis for secular stagnation and discuss the role of equilibrium real rates and the financial cycle in it. Section 3 presents the standard Laubach-Williams model and our expansion thereof to incorporate the financial cycle. In Section 4, the data used for our estimations are explained in detail, while Section 5 presents our empirical results. Section 6 finally concludes.

2. Equilibrium real interest rates, secular stagnation and the financial cycle

The financial crisis of 2008/09 reduced output in leading developed countries considerably. But even after the most severe tensions had been eased, output growth remained consistently lower
than before the crisis. This phenomenon may be explained by the permanent drop in potential output, and has therefore been referred to as “secular stagnation” (Summers, 2014a, 2014b, 2014c, and Teulings and Baldwin, 2014).  

The secular stagnation hypothesis focuses on the real interest rate and its equilibrium value. Under normal circumstances, both should be equalized at the point where aggregate investments equal aggregate savings. However, in a crisis period, and even afterwards, this may no longer be the case. The reason for this is quite simple. While the equilibrium real rate floats freely, the actual real rate faces a lower bound. The latter is due on the one hand to the zero lower bound on nominal interest rates, because individuals can hold excess savings in cash rather than in their bank accounts, thus generating a nominal interest rate of zero. On the other hand, inflation rates or, more precisely, inflation expectations are too low to generate significantly negative real rates. For example, inflation expectations are mainly anchored at about 2 percent in the countries under investigation, being the inflation target of the ECB.  

However, if the equilibrium real interest rate falls below the lower bound of the actual real rate, there is no longer an equilibrium of aggregate investments and savings (Figure 2). So a liquidity trap via excess savings occurs (Crafts, 2015). This permanently lowers the growth rates in the respective country by reducing potential output growth (Teulings and Baldwin, 2014). Secular stagnation is thus a structural problem which has long term consequences, leading, e.g., to permanently higher unemployment rates. While shorter periods of negative real interest rates

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4 In fact, Summers was not the first to detect and make a case for secular stagnation. This term goes back to 1939, when Hansen first developed this theory in what may be considered a similar situation.

5 This being said, one way to significantly lower the actual real rates is to increase the inflation target. For example, Blanchard et al. (2010) proposed increasing the target to about four percent. However, the ECB has not yet changed its inflation target of close to but below two percent in the medium term.
may be tackled by an expansionary monetary policy or fiscal demand-side stimulus, long-term challenges are best met by appropriate supply-side reforms or a combination of supply-side reforms and aggregate demand policies (two-handed approach under hysteresis, Draghi, 2014).

In terms of altered savings and investments, several determinants have been proposed that may have forced economies into secular stagnation since the financial crisis started. On the one hand, these include determinants that only influence savings, for instance, the preference for safe assets, which led to an increase in savings (Caballero and Farhi, 2014), or income inequality (Summers, 2014b). On the other hand, there are determinants that only influence investments, for instance, the degree of innovation (Gordon, 2014, Mokyr, 2014 and Glaeser, 2014), regulation (Jimeno et al., 2014 and Barnes et al., 2013), and declining investment good prices (Summers, 2014b or Glaeser, 2014). Finally, there are determinants such as demographics that affect both savings and investments (Browning and Crossley, 2001, Jimeno et al., 2014 or Gros, 2014).

One other driver of savings and investments is the financial cycle, which affects the ratio of public and private debt to non-financial assets held by households and non-financial corporations (Alcidi, 2017). A high degree of private and public debt relative to assets depresses investment even at low interest rates because individuals and fiscal authorities need to consolidate in the wake of the financial crisis. Moreover, savings are increased to reduce the level of debt. However, supporters of the secular stagnation hypothesis see the key to breaking the vicious circle of permanently lower growth rates in reducing the pressure to consolidate. They tend to propose a lower speed of consolidation and instead favor increasing public investment (Summers, 2014a, Krugman, 2014 and Koo, 2014). In this paper, we implement the financial cycle as proposed by Juselius et al. (2016) in an otherwise standard Laubach-Williams model, which is frequently used to estimate the unobservable equilibrium real interest rate. We explain the way the financial cycle is implemented into the model in the next section.
3. The Laubach-Williams model and the financial cycle

The Laubach-Williams model we use consists of two signal equations and three state equations. All variables are measured as quarterly growth rates. The signal equation (1) is an IS-curve measuring the effect of the first two lags of the real interest rate gap \((r - r^*)\) on the output gap \((Y - \bar{Y})\). Additionally, two lags of the output gap are added to the equation. Equation (2) is the second signal equation, which measures a Phillips curve estimating the influence of the output gap on prices \((\pi)\). Moreover, the prices are assumed to vary with lagged energy prices \((\pi^o)\) since those are a crucial input factor in the production process.\(^6\) Again, lagged values of the dependent variable are added. In this case, and in line with Laubach and Williams (2003), we add eight lags assuming the second to fourth and fifth to eighth lags to have the same influence. Moreover, the coefficients of the lagged inflation rates are restricted to unity, in line with the aforementioned seminal paper.

\[
Y_t - \bar{Y}_t = \alpha_{y,1}(Y_{t-1} - \bar{Y}_{t-1}) + \alpha_{y,2}(Y_{t-2} - \bar{Y}_{t-2}) + \frac{\alpha_r}{2} [(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)] + \varepsilon_{1,t} \tag{1}
\]

\[
\pi_t = \beta_{\pi,1}\pi_{t-1} + \frac{\beta_{\pi,2}}{3} (\pi_{t-2} + \pi_{t-3} + \pi_{t-4}) + \frac{1 - \beta_{\pi,1} - \beta_{\pi,2}}{4} (\pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}) + \\
\beta_y(Y_{t-1} - \bar{Y}_{t-1}) + \beta_o(\pi_{t-1}^o - \pi_{t-1}) + \varepsilon_{2,t} \tag{2}
\]

\[
\bar{Y}_t = \bar{Y}_{t-1} + g_{t-1} + \varepsilon_{3,t} \tag{3}
\]

\[
g_t = g_{t-1} + \varepsilon_{4,t} \tag{4}
\]

\[
z_t = z_{t-1} + \varepsilon_{5,t} \tag{5}
\]

\(^6\) Laubach and Williams (2003) also use import prices as a variable in their Phillips curve specification. We are unable to proceed in that manner here because import price data for most of the Euro-area countries under investigation here are not available for our whole sample period. This would have shortened our sample period considerably, leading to imprecise estimates owing to low degrees of freedom. Garnier and Wilhelmsen (2009) face the same problem when estimating the model for the Euro area. Moreover, Laubach and Williams (2003) added hours worked to their Phillips curve as a robustness check. We also refrain from adding this specification because of data availability and are thus in this respect again in line with Garnier and Wilhelmsen (2009).
The state equations model the time-series generating process of the two unobservable variables, potential output and equilibrium real interest rate. The potential output $\bar{Y}$ is a function of its lagged own value and its unobservable growth rate $g$ (Equation (3)). The growth rate of the potential output is in itself a state variable following a random walk (Equation (4)) as well as the last state variable $z$ (Equation (5)), measuring additional determinants of the equilibrium real rate, such as the time preference of households. The last two equations, (6) and (7), show how the real rate and its equilibrium value are built. In order to save degrees of freedom, the inflation expectations in the real rate are modelled simply by the using adaptive expectations, thus being the lagged inflation rate. This is in line with other studies estimating the equilibrium real rate for the Euro area (Mesonnier and Renne, 2007, Garnier and Wilhelmson, 2009, Belke and Klose, 2013 and 2017, Beyer and Wieland, 2017 or Klose, 2017). The equilibrium real rate is generated in line with Laubach and Williams (2003), representing the sum of trend growth and any additional factors. These additional factors are restricted to have an influence of unity on the equilibrium real rate.

To implement the financial cycle into the standard Laubach-Williams framework, Juselius et al. (2016) propose to expand the model by an additional signal equation covering the leverage gap ($l$) which is expected to be influenced by its own lag the real interest rate gap and the debt-service ratio ($d_{sr}$) as shown in equation (8).

$$l_t = \gamma_t l_{t-1} + \gamma_r [(r_{t-1} - r^*_{t-1}) + (r_{t-2} - r^*_{t-2})] + \frac{\gamma_s}{2} (d_{sr_{t-1}} + d_{sr_{t-2}}) + \varepsilon_{6,t} \quad (8)^7$$

$^7$ Juselius et al. (2016) use only the contemporaneous real interest rate gap in their leverage gap equation as they also do in their IS equation. However, the original Laubach-Williams model uses the first two lags in the latter. That is why we decided to use also the first two lags in the leverage gap equation for reasons of comparability.
Moreover, the IS-equation (1) is modified to include the leverage gap as follows since a high leverage gap should also lower output since the economy tends to invest less and needs to consolidate:

\[
Y_t - \bar{Y}_t = \alpha_{y,1}(Y_{t-1} - \bar{Y}_{t-1}) + \alpha_{y,2}(Y_{t-2} - \bar{Y}_{t-2}) + \frac{\sigma_r}{2}[(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)] + \\
\alpha_l l_t + \varepsilon_{1,t}
\]  

(1a)

However, Laubach and Williams (2013) point out that the error terms in the state equations (4) and (5) are biased towards zero if the model is estimated in one step. This is due to the so-called econometric “pile-up problem” (Stock, 1994). They therefore recommend estimating the model in sequential steps and computing the median unbiased estimator (Stock and Watson, 1998) to solve this problem. We follow this procedure strictly, estimating the model in four steps.

Firstly, both signal equations are estimated separately via OLS to generate reliable starting values. Potential output is proxied by the HP-filter of Y (Hodrick and Prescott, 1997). In the IS equation, the real interest rate gap is omitted at this stage.9

Second, the signal equations are estimated with the Kalman filter, assuming the growth rate of potential output is constant. With these results, we are able to compute the median unbiased estimator \( \lambda_g = \frac{\sigma_4}{\sigma_3} \).

This relationship is used in the third step as a starting point. There we also add the real interest rate gap to the IS equation and model the growth rate of potential output as a time-varying

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The same holds with respect to the debt-service gap where we also use two lags in the leverage gap equation instead of only one as in Juselius et al. (2016).

8 The pile-up problem emerges when pure maximum likelihood methods tend to estimate the standard deviations equal to zero. Given that this is very likely to be the case in our random walk equations (4) and (5), we have to correct for this.

9 This corresponds with the standard procedure according to Laubach and Williams (2003, 2015). For a further practical application see, for instance, Belke and Klose (2017).
variable. Based on these results, we compute the median unbiased estimator for the additional variables affecting the equilibrium real interest rate as \( \lambda_x = \frac{\sigma_z}{\sigma_1} \cdot \frac{\alpha_r}{\sqrt{2}} \).

In the fourth and final step of the standard Laubach-Williams model, we estimate the whole model via maximum likelihood, using the two signal-to-noise ratios.

We have restricted the two coefficients \( \alpha_r \) and \( c \) to lie in the range of -0.3 to 0 and 0.5 to 1.5, respectively. With these restrictions we are well in line with the findings of previous studies where all estimated coefficient parameters fall within these margins.

In order to expand the model by including the leverage gap equation we expand the estimation by two more steps. Since the debt-service ratio is only available for some countries in our study and also only from 1999 onwards, which turned out to be too short a period to generate reliable estimates, we also treat this variable as unobservable, like the potential growth rate and the additional factors.

\[
dsr_t = dsr_{t-1} + \varepsilon_{7,t} \tag{9}
\]

In a fifth step, we therefore add the leverage gap equation (8) and the modified IS-equation (1a) to the standard Laubach-Williams model. In the leverage gap equation, we do not, at this stage, add the debt-service ratio because the pile-up problem occurs for this unobservable variable as well.

Therefore, we use the results of step five to estimate the median unbiased estimator as \( \lambda_x = \frac{\sigma_z}{\sigma_6} \cdot \frac{\gamma_s}{\sqrt{2}} \). Using this parameter, we can estimate the whole model via maximum likelihood. To do so, we also restrict the parameter \( \gamma_s \) to lie in a wide range between 0 and 10. This was the case in all our estimations presented below.
4. The data issue

The standard Laubach-Williams model estimates two unobservable variables: the equilibrium real interest rate and potential output. The method is frequently used to estimate the equilibrium real interest rate.\textsuperscript{10} Also for the Euro area, Mesonnier and Renne (2007), Garnier and Wilhelmsen (2009), Belke and Klose (2013), Beyer and Wieland (2017) and Holston et al. (2016) have used the model to find a measure of the equilibrium real interest rate. Belke and Klose (2017) estimate the model for various Euro-area member countries while Klose (2017) estimates it for several non-Euro-area European member countries. However, all of those papers use some version of the standard Laubach-Williams model.

Our approach expands the model by introducing the financial cycle into it, as Juselius et al. (2016) proposed in a slimmed version for the United States. The financial cycle is introduced here by implementing two additional variables: the leverage gap and the debt-to-service ratio. Since in our case the latter is not available for a period dating far enough into the past to obtain reliable estimates, we model it as an unobservable variable. The leverage gap is modelled in line with Juselius et al. (2016) as the sum of credit to households and non-financial corporations and dividing this by the non-financial assets of the same two groups.\textsuperscript{11} The gap is then calculated by dividing the ratio by its sample mean. The resulting leverage gaps whose changes represent the financial cycle are shown in Figure (2).

- Figure 2 about here –

\textsuperscript{10} See, e.g., Trehan and Wu (2004), Clark and Kozicki (2005), Kiley (2015) or Laubach and Williams (2015) for the US. Holston et al. (2016) estimate the model for the US, Canada, the UK, and the Euro area.

\textsuperscript{11} As a robustness check we also used only the real estate assets. The results are broadly the same as for the non-financial assets. Moreover, we model only private debt while public debt may in principle have the same effects on savings and investments and thus the equilibrium real rate.
In general, the leverage gaps appear to exhibit a positive trend for almost all Euro-area member countries. But especially for Greece and Italy, the economic setback resulting after the financial crisis in the years 2011/12 is clearly visible, since the leverage gap dropped suddenly in value around this time, having increased strongly before.

We estimate the model with respect to nine Euro-area member countries, eight of which were founding members of the Euro area, and Greece, which was the first country to join the monetary union in 2001. Moreover, we estimate the model for the Euro area as a whole. For most of our nine countries and the Euro area in the sample, we obtained quarterly data dating back to 1995. However, for France, the time series date back to 1978, and for Germany to 1991. The end of the sample period is 2015Q4 for all countries under investigation, due to data availability issues.

For each of these countries, we have collected the data on real GDP, consumer prices, energy prices, and interest rates besides the leverage gap explained above. All data are seasonally adjusted and taken from the OECD database. As the relevant interest rate, we use the three-month interbank rate in line with several of the other studies in this field cited above. Since the countries in question have not had their own interbank rates since the Euro area was established, we added the data from the three-month EURIBOR for all dates where each respective country was a member of the monetary union, that is, starting in 1999 for the eight founding members and in 2001 for Greece.

Our interpretation of the results is based on a comparison of the estimated equilibrium real rate and the observed real rate. For this purpose, we make use of two concepts in measuring the

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12 Please note that this positive trend does not cause any estimation problems because the leverage gap enters into the model with its quarterly growth rate which appears to be stationary for all countries.

13 Austria, Belgium, Finland, France, Germany, Italy, Luxembourg, and the Netherlands. For the remaining three founding members, Ireland, Portugal, and Spain, we were unable to calculate the leverage gap due to missing data. The other seven countries that have now adopted the Euro were also excluded due to data non-availability.
latter: ex-ante and ex-post real rates. The former represents the nominal interest rate minus the expected inflation, which in our case is specified as the lagged inflation rate according to the adaptive expectations hypothesis \( r_t = i_t - \pi_{t-1} \), while the latter is formulated as the interest rate minus the observed inflation rate until maturity \( r_t = i_t - \pi_t \).\(^{14}\) Even though the estimated real interest rates turn out to differ depending on the concept used, this will have only a minor influence on the results, i.e., whether or not we are able to conclude that secular stagnation may be a relevant problem in a Euro-area member country.

5. Results

Our results of the standard Laubach Williams model (Table 1)\(^{15}\) indicate that the coefficients are generally in line with previous studies. This holds also for the restrictions set on some coefficients. However, especially with respect to \( \alpha_r \), the influence of the real interest rate gap on output, we are unable to obtain significant parameter estimates. However, other studies (Mesonnier and Renne, 2007, Garnier and Wilhelmsen, 2009, and Belke and Klose, 2017) encountered similar problems when estimating the model. Only for France do we indeed find a significantly negative coefficient. But the point estimates, which are about -0.15 in most of the cases, are very stable over the various countries. With respect to the parameter \( c \), which represents the influence of potential growth on the equilibrium real rate and is inserted into our IS equation, we find significant estimates for two countries (Finland and Greece). Moreover, the point estimates vary widely, with a range of 0.5 to 1.5, although none of these exceeds the boundaries we set at the end of Section 3. Our median unbiased estimators \( \lambda_g \) are generally in line with estimates for other countries in previous studies. The estimate of \( \lambda_z \) is, however,

\(^{14}\) See also Hamilton et al. (2015) upon this issue.

\(^{15}\) Only the final estimates of the fourth step are presented here. The results for the previous steps are available from the authors upon request.
slightly lower.\textsuperscript{16} But the remaining parameter estimates and variances are well in line with other studies in this field. Thus, we feel justified in concluding at this stage that the parameter estimates are generally comparable with those from other studies.

- Table 1 about here -

Expanding the model to include the financial cycle, the coefficient estimates does not change much in most cases; thus, our results remain robust (Table 2). Moreover, most of the coefficients associated with the leverage gap show the expected signs, i.e., a negative response of the leverage gap in the IS equation and a positive influence of the real rate gap in the leverage gap equation, although the coefficients remain insignificant in the majority of the cases. The restriction imposed on the debt-service ratio proves to be valid, with only two countries coming close to the lower bound of zero but actually not meeting it. We can therefore also conclude for the expanded model including the financial cycle that coefficient estimates are generally plausible.

- Table 2 about here -

5.1. Output gap

The Laubach-Williams model estimates not only the equilibrium real interest rates but also the potential output, thus allowing the output gap to be estimated. Here we have generated two estimates of the output gap: The first is based on the standard Laubach-Williams model and the second on the extended model, incorporating the leverage gap in the IS equation (Figure 3). The output gaps are based on the predicted or one-sided estimates. However, the results for the

\textsuperscript{16} Please note that we explicitly estimate the median unbiased estimator. Other studies in this field so far (Mesonnier and Renne, 2007, Garnier and Wilhelmsen, 2009, or Beyer and Wieland, 2017) had to restrict this coefficient to obtain reasonable results.
smoothed or two-sided estimates present a similar picture, and are available from the authors upon request.

- Figure 3 about here -

Thus, the output gaps should differ because of this different setting. However, for the Euro area as a whole, and most Euro-area member countries, the output gaps are broadly comparable when the financial cycle is included in the model. Only for Austria, Belgium, and Germany do the output gaps tend to increase when the financial cycle is included, while the output gap decreases when the model is augmented with the financial cycle in the case of Luxembourg. The latter is possibly due to important role of the financial industry in Luxembourg.

5.2 Ex-ante real interest rates

Concerning the estimates of the equilibrium real interest rate, we first compare them to the ex-ante real rates. The one-sided (predicted) estimates deliver some interesting results (Figure 4).

- Figure 4 about here -

First, for the Euro area as a whole, the equilibrium real interest rate incorporating the financial cycle in the Laubach-Williams model leads to slightly lower rates than in the standard model. Our findings for the Euro area therefore differ from those of Juselius et al. (2016), who estimated higher equilibrium rates for the United States when the financial cycle was added. However, the real interest rate is even lower in these cases. Hence, there is no indication that secular stagnation has been a problem in recent years, even if the financial cycle is taken into account. The same holds for Austria and Luxembourg.

Second, for some countries, the equilibrium real rate tends to be higher when the financial cycle is added to the model. This holds for Belgium, Italy, and the Netherlands. But for Italy and the Netherlands, the actual real rate was even lower than the equilibrium real rate excluding the
financial cycle. So with our extended model, for these countries, it becomes even clearer that they are not facing a period of secular stagnation. However, for Belgium, both equilibrium real rate estimates are clearly negative at the end of the sample period and thus even lower than the actual real rate, suggesting that secular stagnation may be a problem in this case from a Bayesian viewpoint. It must be admitted, however, that the estimation uncertainty surrounding the equilibrium real rate estimates is quite large, making it difficult to identify significant differences from the actual real rate. This holds not only for Belgium but also for all other countries.

Third, for the remaining countries, there is no clear inference whether the equilibrium real rate estimated with the model including the financial cycle is higher or lower. For some countries (Germany, Greece), the estimates are indeed quite similar, while for others (Finland, France), the estimates with the extended model are below the standard model estimates in one period and above in other periods. Especially with respect to France, the equilibrium real interest rate seems to be more volatile when the financial cycle is added to the model. At the end of the sample, both estimates of the equilibrium real rate are more or less equal at about -5 percent and thus clearly below the actual real rate, which may cause problems of secular stagnation. The same also holds for Greece, where both equilibrium real rate estimates are considerably lower than the actual real rate; thus, secular stagnation may be present in this country as well.

*Figure 5 about here*

The results are reinforced further when the two-sided (smoothed) estimates are considered (Figure 5). The same trends can be identified in the equilibrium real rate with and without the
financial cycle. Moreover, there are concerns whether Belgium, France, or Greece face secular stagnation, but these exist irrespective of the model used.\footnote{Please note that Belke and Klose (2017) found only Greece to be facing problems of secular stagnation in a standard Laubach-Williams model that was more or less the same as ours. However, the results differ for Belgium and France because of the considerably shorter sample period due to limitations in data availability with respect to the financial cycle.}

5.3 Ex-post real interest rates

When we employ ex-post realized real interest rates instead of ex-ante rates, the estimates for the equilibrium rates remain almost unchanged: They are only shifted backwards by four quarters, but the estimates themselves remain the same (see Figure 6 for the one-sided time-series, and Figure 7 for the two-sided estimates). However, the empirical realizations of the real interest rates might be different. While we observe that the values are indeed different from those obtained with the ex-ante data, the overall results remain robust. Hence, our observations in the previous section regarding the differences between the two equilibrium real rates estimated with our two models are still valid. Only Belgium, France, and Greece therefore face problems concerning secular stagnation.

- Figures 6 and 7 about here -

6. Conclusions

In this paper, we have investigated whether the Euro area as a whole or individual Euro-area member countries face a period of sustained lower economic growth, a phenomenon known as secular stagnation. We have addressed this question by estimating equilibrium real interest rates in the Euro area, employing the Laubach-Williams method, and comparing them to actual real rates. Since the financial crisis has altered the degree of leverage in several European economies, we have expanded our model to incorporate the financial cycle.
We have estimated the models with respect to the Euro area as a whole and individually for nine Euro area member countries. We have shown that incorporating the financial cycle indeed changes the estimated equilibrium real interest rates. For a few Euro-area member countries, our estimates of the equilibrium real interest rate are clearly higher than the standard ones. We thus show that attributing the decline in real interest rates primarily to an exogenously caused reduction in the natural interest rate is inadequate and does not take financial factors into account. In other cases, including our estimates for the Euro area as a whole, the estimated equilibrium real rates are slightly lower than without taking the financial cycle into account, but are still higher than the actual rates. This indicates that real monetary policy rates were set even more systematically and consistently below (or not as far above) the natural real rate. Comparing the sequence of actual and equilibrium real rates, only Belgium, France, and Greece may potentially be facing a period of secular stagnation. Even though incorporating the financial cycle changes the estimated equilibrium real interest rate considerably in some cases, the inference of secular stagnation thus remains overall robust for nearly all countries.

However, some caveats to the financial cycle-augmented model introduced by Juselius et al. (2016) should be noted that point to areas requiring further research: First, in the current setting, the leverage gap is added to the model as an additional explanatory variable, which is influenced by the real rate gap and thus the equilibrium real rate. But our remarks in Section 2 suggest that the mechanism may also work the other way around, i.e., leverage may determine the equilibrium real interest rate. Therefore, the state equation forming the additional factors could be expanded by the leverage gap.

Second, this study uses only private-sector leverage data. However, public debt is also a problem especially in several southern European countries. Therefore, the analysis could be expanded to incorporate public debt or leverage as well.
Third, statistical inference regarding secular stagnation is difficult due to the large confidence bands around our estimated equilibrium real rates. This holds irrespectively of whether the standard Laubach-Williams model or the financial cycle-augmented version is used. The main reason for this is the short sample period, starting in 1995 for most of the countries. Therefore, more (and earlier) data should be collected to allow for more robust inferences.

References


Blanchard, O., Dell'Ariccia G. and Mauro, P. (2010): Rethinking Macroeconomic Policy, IMF Staff Position Note 10/03.


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Notes: ML-estimation; EA=Euro-Area, AT=Austria, BE=Belgium, DE=Germany, FI=Finland, FR=France, GR=Greece, IT=Italy, LU=Luxembourg, NL=Netherlands; standard errors in parenthesis; ***/***/* means significance at the 1%/5%/10% level.
Table 2: Parameter estimates Laubach-Williams model including financial cycle

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| Variance | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ | $\sigma_5$ | $\sigma_6$ | $\sigma_7$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\log -$ likelihood |
|---------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|----------------|---|
| $\sigma_1$ | 0.2531     | 0.0000     | 0.1191     | 0.5740     | 0.0000     | 0.0000     | 0.0333     | 0.0000     | 0.2720     | 0.0000     | 0.0000     | -116.85   |
| $\sigma_2$ | 0.0612     | 0.0779     | 0.1729     | 0.0815     | 0.1289     | 0.0925     | 0.0205     | 0.0496     | 0.1305     | 0.0814     | 0.2377     |
| $\sigma_3$ | 0.0132     | 0.2713     | 0.0113     | 0.0000     | 1.6125     | 0.0720     | 1.1498     | 0.0000     | 2.0564     | 0.0000     | 0.0000     |
| $\sigma_4$ | 0.0001     | 0.0002     | 0.0002     | 0.0000     | 0.0014     | 0.0003     | 0.0001     | 0.0000     | 0.0042     | 0.0000     | 0.0000     |
| $\sigma_5$ | 0.0002     | 0.0000     | 0.2399     | 0.4304     | 0.0000     | 0.3758     | 0.0000     | 0.0013     | 0.0000     | 0.0000     | 0.0000     |
| $\sigma_6$ | 0.3952     | 1.7514     | 1.1639     | 0.4792     | 1.9006     | 1.1476     | 1.5950     | 4.4408     | 17.5976     | 4.6361     |
| $\sigma_7$ | 0.0004     | 0.0000     | 0.0000     | 0.0000     | 0.0000     | 0.0006     | 0.0321     | 0.0000     | 0.0002     | 0.0000     | 0.0000     |
| $\lambda_1$ | 0.0166     | 0.0275     | 0.1218     | 0.0124     | 0.0026     | 0.0628     | 0.0006     | 0.0277     | 0.0452     | 0.1049     |
| $\lambda_2$ | 0.0006     | 0.0006     | 0.0453     | 0.0006     | 0.0006     | 0.0280     | 0.0006     | 0.0006     | 0.0006     | 0.0006     |
| $\lambda_3$ | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     |
| $\lambda_4$ | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     | 0.0006     |

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Figures

Figure 1: Real interest rates and secular stagnation

Notes: \( i \) = real interest rate, \( I \) = aggregate investments, \( S \) = aggregate savings, approximately -2 percent as intercept is chosen because the minimum should be given at a zero nominal rate minus the ECB inflation target of about two percent.
Figure 2: Leverage gap for the Euro area and nine Euro-area countries
Figure 3: Output gap

Notes: Output gap standard Laubach-Williams model = red line, output gap with financial cycle = black line.
Figure 4: Predicted real and equilibrium real interest rate ex-ante

Notes: Real interest rate = blue line, equilibrium real interest rate standard Laubach-Williams model = red line, equilibrium real interest rate with financial cycle = black line.
Figure 5: Smoothed real and equilibrium real interest rate ex-ante

Notes: Real interest rate = blue line, equilibrium real interest rate standard Laubach-Williams model = red line, equilibrium real interest rate with financial cycle = black line.
Figure 6: Predicted real and equilibrium real interest rate ex-post

Notes: Real interest rate = blue line, equilibrium real interest rate standard Laubach-Williams model = red line, equilibrium real interest rate with financial cycle = black line.
Figure 7: Smoothed real and equilibrium real interest rate ex-post

Notes: Real interest rate = blue line, equilibrium real interest rate standard Laubach-Williams model = red line, equilibrium real interest rate with financial cycle = black line.